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Isolated exchange with interpersonal comparison of utility: a special hypothesis

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ISOLATED EXCHANGE WITH INTERPERSONAL COMPARISON OF UTILITY:

A SPECIAL HYPOTHESIS

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I. INTRODUCTION

Isolated exchange may be defined as a situation in which two or more persons, which have each a certain amount of commodities, try to exchange these commodities with each other, in order to increase the utility for each person belonging to the amount of commodities held by that person. If only two persons are involved, isolated exchange is sometimes called bilateral monopoly.

One of the interesting and at the same time disturbing aspects of isolated exchange is, that persons need to reach agreement with each other about the particular amount of commodities to be exchanged. Each person which is a party in the isolated exchange, will consider in general several factors, which will determine for him which solution is reasonable and which not.

In this thesis we will not consider all the factors having influence on the solution of the isolated exchange situation. Instead we will concentrate only on the possible effect of one particular factor, namely the effect of the utilities belonging to the commodities of each person on the solution, if these utilities can be compared by all persons. In short, we are interested in the effect on the solution if interpersonal comparison of utility is possible.

There are many solutions proposed for the case of isolated exchange and the situation would become rather complicated if the effect of interpersonal comparison of utility would be considered for all these solutions. As a main source for the differences in solutions however, may be considered the differences in bargaining skills of the persons. If we exclude the effects of differences in bargaining skills therefore, the number of possible solutions may be greatly reduced and consequently the investigation may become much simpler.

One particular aspect of bargaining skills is the ability of a person of misinforming other persons about one's own utilities. Misperceptions about utilities may influence the solution. We will exclude these effects by assuming complete information of all persons.

We will find it convenient to consider the process in which the persons try to reach agreement, as a special type of repeated play of a game. On the one hand the game will represent all the aspects of the isolated exchange situation and on the other hand it will be relatively easy to conduct the game as an experiment to test the proposed solution.

As a starting point we will consider in Chapter II the competitive solution and the Nash solution and make a comparison between them. In Chapter III we will state first explicitly the conditions under which we will investigate the isolated exchange situation. Based upon these conditions we will then criticize the competitive solution and the Nash solution and propose and criticize a special hypothesis for a solution.

In Chapter IV we consider in particular the two person isolated exchange situation and generalize the hypothesis. Finally in Chapter V we consider the three person isolated exchange situation and the complications which are introduced by the increase in number of persons from two to three.

II. THE COMPETITIVE SOLUTION AND THE NASH SOLUTION

A. The Static Theory of Pure Exchange

As Hicks (2, p. 1) pointed out in the introduction of "Value and Capital", the mathematical method makes it possible to deal with different economic subjects in one book, by the unity in method. He showed this subsequently by integrating the theory of exchange and the theory of production.

Since the publication of his contribution in 1939, it has gradually become customary to use the same approach even if the subjects are dealt with separately. The elegancy of the method may be demonstrated perhaps by the fact that we can describe the multi-market equilibrium conditions in a static pure exchange economy of n individuals and m commodities by the following set of equations (1, p. 133):

$$E_{ij} = E_{ij}\left(1,\frac{p_2}{p_1},\ldots,\frac{p_m}{p_1}\right) \qquad \begin{matrix}(i = 1,\ldots,n)\\(j = 1,\ldots,m)\end{matrix} \qquad 1$$

$$\sum_{i=1}^{n} E_{ij} = 0 \qquad (j = 1,\ldots,m) \qquad 2$$

The equations 1 are mn individual excess demand functions, which are homogeneous of degree zero in prices. The m conditions 2 state that every market must be cleared. The system contains mn + m equations with the mn individual excess demands and the (m - 1) exchange ratios as variables. This implies that one of the equations is functionally dependent upon the others, so that the system can only be solved for relative prices. The quantities and prices satisfying the equations 1 and 2 we call for future reference the competitive solution.

The individual excess demand functions 1 can be derived from the first order conditions which must be satisfied by the consumer's constrained utility index:

$$V_i = U_i(E_{i1} + q_{i1}^o, \ldots, E_{im} + q_{im}^o) - \lambda(\sum_{i=1}^{m} p_j E_{ij}), \qquad 3$$

provided that the second order conditions are also satisfied (1, p. 130).

The major drawback of this mathematical set up is, that it is a static equilibrium theory. It can only consider situations where the equilibrium conditions, expressed by the first and second order conditions on 3, are fulfilled or almost fulfilled for infinitesimal small departures from the equilibrium solution.

B. An Adjustment Mechanism

As the equilibrium itself is a special situation, which may be reached at the end of an exchange process, rather than be the starting point of it, many writers have tried to give more attention to the process by which the equilibrium may be reached. In fact this had been the traditional approach before the equilibrium theory by Hicks and others was developed. As an example of this approach we will consider "The Theory of Exchange", by Peter Newman (6).

In Chapter IV, Newman describes the attainment of equilibrium in bilateral exchange by considering an adjustment mechanism and its convergence. We will summarize his arguments.

In Figure 1 we have shown in an Edgeworth box diagram similar to those used by Newman, the necessary aspects to describe the working of the adjustment mechanism. At the beginning of the exchange the positions of

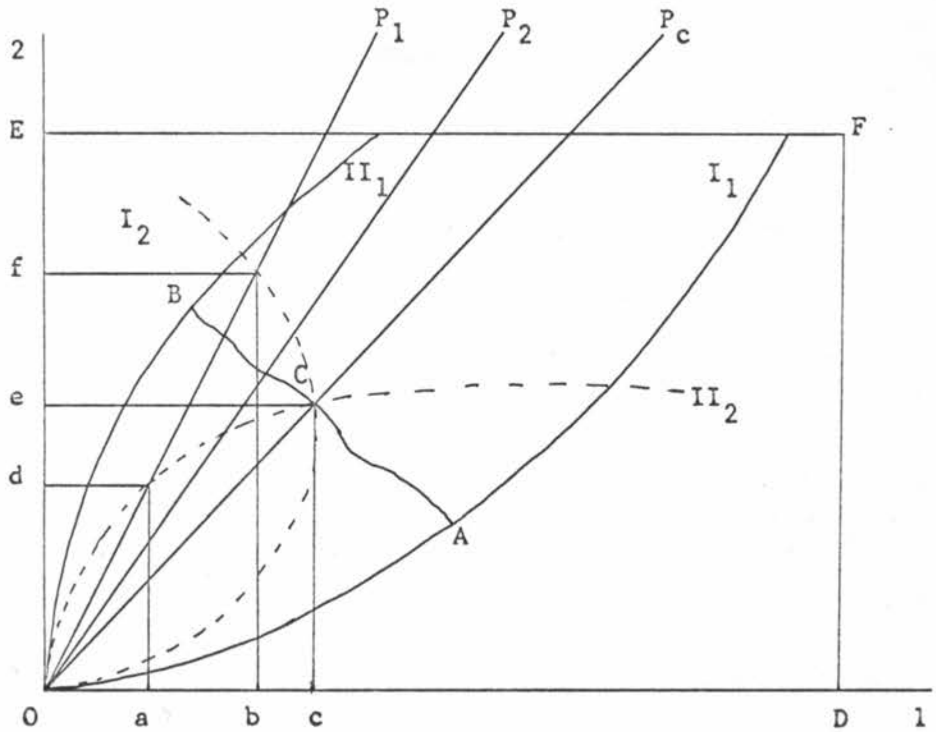


Figure 1. Edgeworth box diagram for persons I and II and commodities 1 and 2

the two persons I and II is at the point O. Person I has only the amount OD of commodity 1 and person II has OE of commodity 2. The point O should not be interpreted as origin. Strictly speaking an Edgeworth box diagram has always two origins, one with respect to every person. In Figure 1, the point D is the origin with respect to the amounts of commodities of person I and E is the origin with respect to the amounts of commodities of person II.

The set of combinations of commodities 1 and 2 for which the utilities to person I are the same as the utility of the amount OD of commodity 1, is called the initial indifference curve of person I. It is a curve through O convex to the point D and shown as I_1 in Figure 1. Likewise the indifference curve belonging to the utility of the initial amount OE of commodity 2 for II, is shown as the curve II_1 through O convex to E in the figure. The contract curve is the set of points inside of the indifference curves which is such, that for any point outside the set but which is within the closed region bounded by the indifference curves belonging to the initial states of persons I and II, there is at least one point in the set which is preferred by both I and II. Furthermore no point on the contract curve is preferred by both over any other point on the contract curve. The contract curve is also called the set of Pareto optimal points. It is shown as the curve AB. It may be found as the set of points of tangency of the indifference curves of persons I and II within the region bounded by the initial indifference curves.

We have also shown the trading curve of each person. The trading curve of person I is indicated by I_2 and the trading curve of II by II_2.

A trading curve of a person is the set of solution points to the bilateral exchange situation, at which that person maximizes his utility, given all possible exchange ratios such that no negative entries enter in the exchange ratios or prices. The last condition means that the angle of the priceline with the positive x-axis must not be smaller than 0° and not greater than 90°.

In order to find the trading curves we need the indifference curves belonging to all possible points, at least within the region enclosed by the indifference curves belonging to the initial amounts of commodities of persons I and II. At the price P_1 for instance, we find the point on the trading curve of person I, by maximizing person I's utility given that price and supposing that the point at which he maximizes his utility is a solution point of the exchange situation. From the equilibrium theory dealt with before, we know that the point on the trading curve will be the point of tangency of the priceline with an indifference curve of person I. This point is indicated by the point with coordinates b and f in the figure. In the same way we find the point on the trading curve of II for the same price P_1, as the point indicated by the coordinates a and d.

We can now describe the working of the adjustment mechanism. It consists of a control mechanism, the price and a decision rule which determines the direction of adjustment of the price, given the bids of persons I and II in the market. The adjustment process works as follows. Suppose the price in the market at a certain moment is P_1. Then both persons I and II will take this price as given and offer an amount for exchange such that they will both maximize their utility. That is, they will offer amounts for exchange determined by the intersection of the priceline and their trading curves.

So at P_1, person I will offer b of commodity 1 in exchange for f of commodity 2. Person II will offer d of commodity 2 in exchange for a of commodity 1. Apparently at the price P_1 the amounts of each commodity offered and demanded do not match. There is an excess offer of commodity 1 and an excess demand for commodity 2 and exchange will not take place. The decision rule now determines that the price will be adjusted in a direction favorable to the commodity with excess demand. In our case this is commodity 2. In terms of Figure 1 this implies that the angle of the priceline with the positive x-axis will become smaller. Say the new price is P_2.

At the new price, persons I and II will make again an offer determined by the intersection of price and trading curves. If the bids still do not match, the price will be adjusted again in a direction favorable to the commodity in excess demand. This process will be repeated until there is no excess demand for one commodity and consequently also not for the other commodity. It is clear that the process will finish at the point where the trading curves intersect. As at this intersection no other point is preferred by both parties, this intersection must be on the contract curve. It is indicated by the point C in the figure.

At the intersection also the conditions 1 and 2 of page 3, are precisely satisfied for the case $n = 2$, $m = 2$. Newman shows subsequently under what conditions we may expect that solutions may exist and are unique. The solution will always exist if the area enclosed by the indifference curves consits of more than one point, but it need not be unique. The case of non-uniqueness is however quite special and depends on rather special shapes of the indifference curves. In general therefore we may assume the solution to be unique.

As the solution satisfies conditions 1 and 2 of page 3, we will call it also the competitive solution. As the competitive solution indicated by point C lies on the contract curve, we have the property that a competitive solution is Pareto optimal. On the other hand not any point on the contract curve is the competitive solution. Therefore the fact that a point is Pareto optimal does not imply that it is the competitive solution.

C. An Alternative Derivation of the Competitive Solution

The device of the Edgeworth box diagram has some advantages which makes it particularly useful in bilateral exchange situations. A disadvantage however is, that the device can not be extended to deal with three person situations. In this section we will derive the competitive solution of the two person two commodity exchange situation by a method which is equivalent to the approach used in section B, but lends itself more easily for cases of more than two persons.

Let us therefore consider again Figure 1. But now we want to consider the point 0 explicitly as the origin of the space of commodities 1 and 2. Along the x-axis we measure the amount of commodity 1 and along the y-axis the amount of commodity 2. Furthermore we indicate by I the vector of amounts of commodities 1 and 2, person I has initially. In the same way II indicates the amounts of commodities 1 and 2 person II has initially. We will use interchangeably the term vector I, or point I, or position of I.

Suppose persons I and II have initially the same amounts as they had in Figure 1. Figure 2 then shows the positions of persons I and II in the commodity space. The vector (c,o) for instance has as first coordinate the amount c of commodity 1, which is equal to the amount OD in Figure 1. The total amount of commodities of persons I and II together we indicate by the vector $T_{(I,II)}$. Persons I and II may change their positions by exchanging with each other. Whatever exchange will take place, the total amount of commodities indicated by the sum of the new positions of I and II will always add up to $T_{(I,II)}$.

We now construct the indifference curve belonging to the initial position of person I, starting from I instead of from O and convex to the origin O. The position of the indifference curve of person I with respect to the line segment from I to $T_{(I,II)}$ in Figure 2, is equivalent to the position of the indifference curve of person I with respect to the line segment OE in Figure 1. In the same way we construct the indifference curve of II, which is also convex with respect to the origin O. The initial indifference curves of persons I and II are indicated respectively by I_1 and II_1.

For each person there exists an infinite number of indifference curves, belonging to all possible positions in the commodity space. If we exclude negative amounts of commodities, all possible positions of

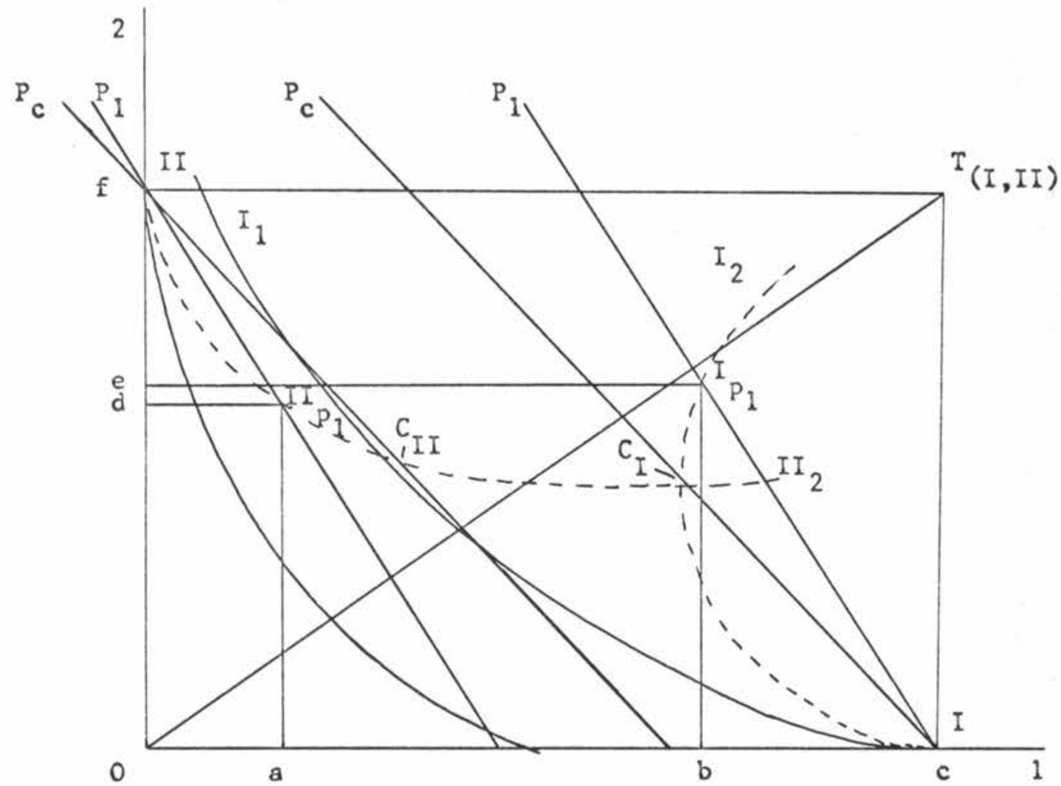


Figure 2. Exchange diagram for persons I and II and commodities 1 and 2

persons I and II must be within the box $OIT_{(I,II)}II$. At any position of a person an exchange ratio or price is indicated by a straight line through the position of the person. In general therefore an exchange ratio is indicated by two parallel straight lines, each through the position of one person. In the figure we have indicated an exchange ratio by the line P_1.

The trading curves of persons I and II we may find, by the same method as in section B, as the sets of points of tangency of any price line with the indifference curves of the person. We have shown possible trading curves of persons I and II by I_2 and II_2. According to the same procedure as in section B, person I will at the price P_1 offer an amount for exchange such that he reaches his trading curve. At the price P_1 for instance person I will offer an amount (c - b) of commodity 1 in exchange for an amount e of commodity 2. On the other hand II will offer at the same price (f - d) of commodity 2 for a of commodity 1.

The new positions of persons I and II we have indicated by I_{P_1} and II_{P_1}. But at these new positions the sum of the vectors I_{P_1} and II_{P_1} does not add up to $T_{(I,II)}$. This combination of positions therefore is impossible, exchange does not take place because the market is not cleared.

We may now use the same device of the adjustment mechanism to find the competitive solution. The price will be adjusted in a direction favorable to the commodity in excess demand until the sum of the new positions of persons I and II adds up to $T_{(I,II)}$. This is also exactly at the point where the sum of the displacements of I and II adds up to zero. In the figure we have shown the solution at the price P_c for person I as C_I and for II as C_{II}. The sum of the vectors $(C_I - I)$ and $(C_{II} - II)$ is zero. Furthermore it can be shown that the amounts exchanged at the competitive solution are identical with the amounts exchanged at the competitive solution in Figure 1.

So instead of being one point, the competitive solution now consists of a point for each person. The same will be true in general of the Pareto optimal curve. To see this let us look at Figure 3, which is the same as Figure 2 as far as the positions of persons I and II and the indifference curves are concerned.

Suppose person II moves to the point D on his own indifference curve. Then, in order for exchange to be possible, person I has to move to B. The vectors D and B add up exactly to $T_{(I,II)}$. If we let move person II along his initial indifference curve we can trace out the curve along which I has to move in order for exchange to be possible. This curve of person I we have indicated by I_3. Person I can never get more to the right for any possible position of II on his initial indifference curve, as indicated by I_3, because any position to the right of the curve I_3 implies a position of II to the left of his initial indifference curve i.e., a position worse than the original position of person II. It is not reasonable to expect that person II will agree upon such an exchange. We may call the curve I_3 therefore, the curve of maximum possible utility gains of person I for any position of II along his initial indifference curve.

Likewise we may construct the curve II_3 of maximum possible utility

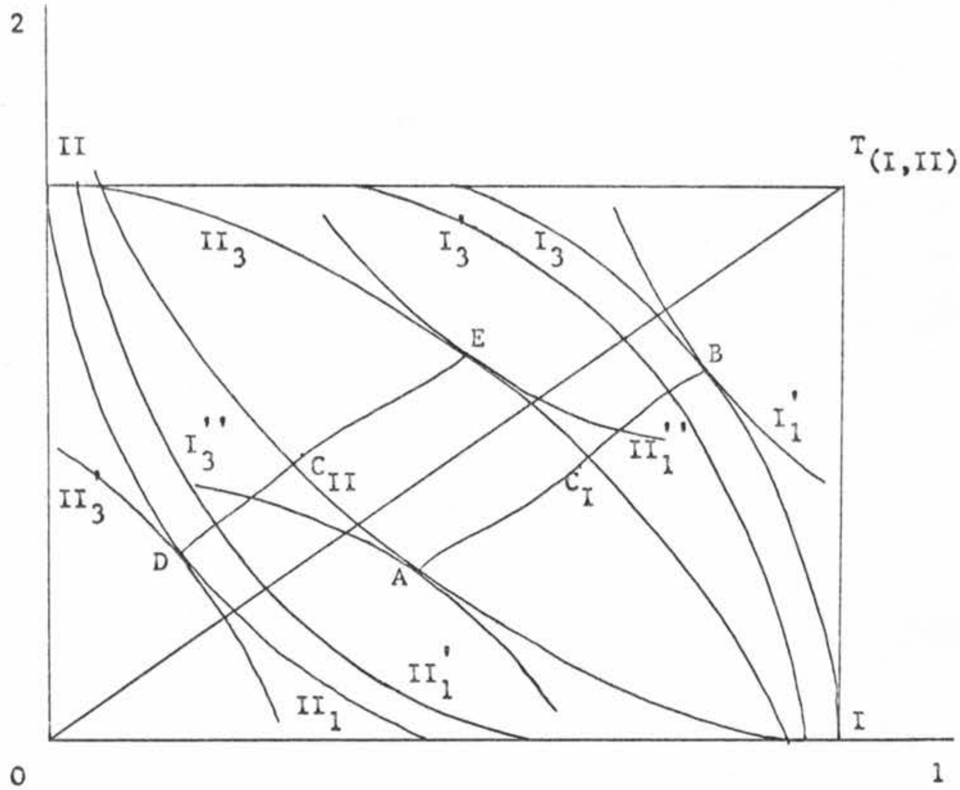


Figure 3. The curves of maximum possible utility gains and the contract curves for persons I and II

gains of II for person I moving along his initial indifference curve. But the initial indifference curve is not the only indifference curve a person may move along. Suppose for instance we let person II move along his indifference curve II_1'. Then person I has to move along I_3' in order for exchange to be possible.

Suppose now we let person II move along the indifference curve II_1'', which is tangent to his curve of maximum possible utility gains II_3 at E. Then person I must move along a curve I_3'' which is tangent to his initial indifference curve, say at A. Because as the curve II_1'' has only one point in common with the curve II_3, there can only be one point for person I on the curve I_1 by the property that for any vector in the commodity space there is exactly one other vector such that their sum adds up to $T_{(I,II)}$.

At E the utility gain of II is at the maximum while at A the utility gain of person I is zero. The points A and E are a Pareto optimal combination of points for persons I and II. That is, it can be shown that no movement away from the points A and E is possible with the consent of both players. If person II for instance moves along his indifference curve II_1'' at E, person I has to move along I_3''. This implies that if person II remains at the same utility level or increases his utility level, person I will decrease his utility level in order for exchange to be possible. On the other hand, if person I remains at his utility curve I_1 at A, person II must remain at II_3, which implies a decrease in utility for person II.

Likewise it can be shown that if person I remains at his indifference curve I_1', which is tangent to I_3 at B, then II has to remain at II_3' which is tangent to II_1 at D. The points B and D again are a Pareto optimal combination of points for I and II.

By tracing out all points of tangency of indifference curves with curves of maximum possible utility gains for each person, we will find the curve of Pareto optimal positions for each person. In general only the set of Pareto optimal positions belonging to feasible solutions is of interest. A feasible solution is a solution of the exchange situation such that no party looses utility. In Figure 3 we have shown the feasible set of Pareto optimal positions for person I as the curve AB and for person II as DE. We may call these curves the contract curves of the persons.

As the competitive solution is Pareto optimal, it must lie on these contract curves. We have shown the competitive solution for person I as C_I and for person II as C_{II}. The points C_I and C_{II} are identical with those of Figure 2.

The method used in this section to derive the competitive solution therefore, may be considered equivalent to the method used in the previous section. It enables us to derive all the information we can derive also from the previous method. The Edgeworth box diagram exhibits one Pareto optimal curve but two origins. The present method exhibits one origin but two Pareto optimal curves. The present method however, can be extended to cases of more than two persons as we will show in later sections.

The contract curves of both persons need not always be different. If certain conditions are satisfied, they may be the same for both persons. We will show one such particular case because it still can show the aspects in which we are interested.

Suppose persons I and II have the same indifference curves, which are orthogonal hyperbolas. Suppose further the initial positions of persons I and II are on the same indifference curve and symmetric with respect to the straight line k from the origin with angle of 45° with the x-axis.

In Figure 4 we have shown the particular positions of persons I and II on their initial indifference curve $I_1 = II_1$. The total amount of commodities is represented by the vector $T_{(I,II)}$. The trading curves of persons I and II are also symmetric with respect to the line k and indicated by I_2 and II_2. They intersect at the point C. At C the diagonals of the parallelogram $OIT_{(I,II)}II$ intersect each other perpendicular. C is the competitive solution for this particular exchange situation.

The curves of maximum possible utility gains of persons I and II, if the other person remains on his initial indifference curve, are identical. They are indicated by $I_3 = II_3$.

We now assert that the contract curves of persons I and II are the same and are indicated by the straight line segment AB of k. To show this we may remark first that a property of orthogonal hyperbolas is, that every straight line from the origin intersects the hyperbolas successively at points which have all parallel tangent lines.

The straight line k from the origin intersects the indifference curve $I_1 = II_1$ at A. The tangent line at A we have indicated by p. If person I is at A, person II has to be at B, which is also on the line k and on $I_3 = II_3$, in order for exchange to be possible. At B the line k

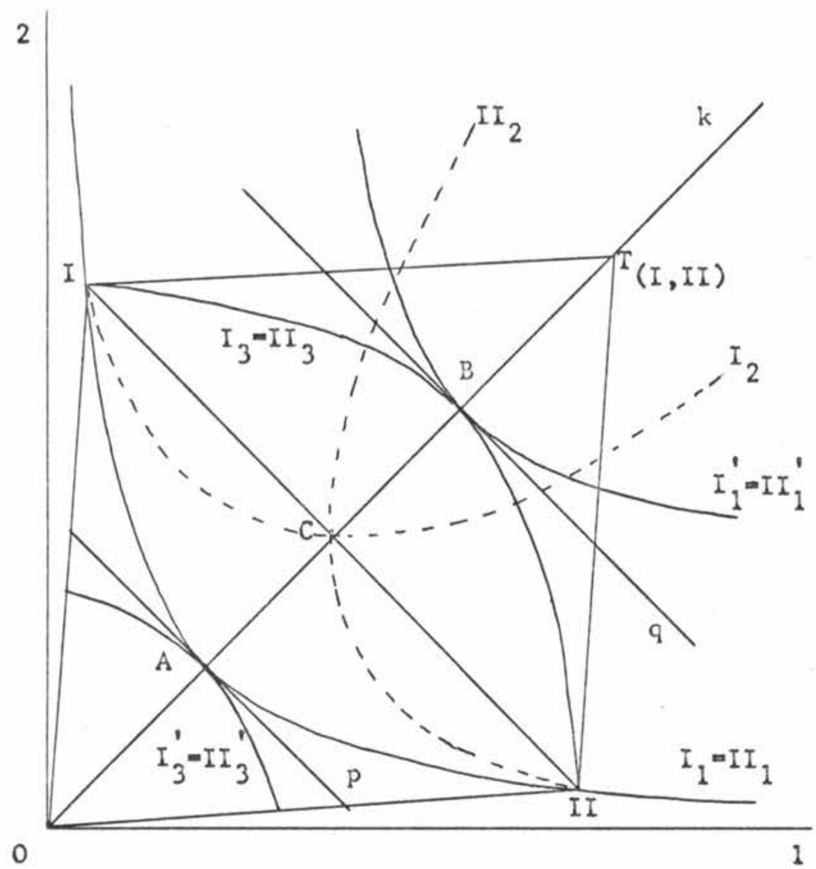


Figure 4. A special symmetric exchange situation

intersects an indifference curve say $I_1' = II_1'$. The tangent line at B, which is parallel to p, we have indicated by q. If person I moves along p, person II has to move along q. As p does not intersect $I_1 = II_1$, q can not intersect $I_3 = II_3$. Therefore at B the indifference curve $I_1' = II_1'$ is tangent to the curve $I_3 = II_3$.

On the other hand if person II moves along $I_1' = II_1'$ at B, person I must move along $I_3' = II_3'$ at A. For the same reason $I_3' = II_3'$ is tangent to $I_1 = II_1$ at A. Therefore the points A and B on the line k form a combination of Pareto optimal positions for persons I and II.

Likewise it can be shown that for any position of person I on the straight line between A and B, person II will have also a position on the straight line between A and B, such that the combination of positions again is Pareto optimal. Furthermore for any position of person I in the region IAIIB but not on the straight line AB, the position of person II will also not be on the line segment AB. In that case there will always be at least one point for each person on the straight line AB, such that the utility of at least one person is increased and such that exchange is possible. Therefore the straight line AB is the contract curve for both persons I and II.

Only at the competitive solution point C, the positions of both persons on the contract curve are at the same point.

D. Nash's Solution to the Bargaining Problem

A characteristic which the adjustment mechanism, considered in the previous sections shares with the equilibrium theory approach, is that the parties take the price of the commodities as given and adjust the quantities given any price. This is certainly not the only possible way parties can behave in isolated exchange situations. A solution which may be reached in the case both parties adjust prices as well as quantities, is proposed by Nash (4).

In "The Bargaining Problem", Nash investigates the behavior of two persons which "have the opportunity to collaborate for mutual benefit in more than one way". The economic situation of bilateral monopoly may be considered as a bargaining problem, but also the case of isolated barter exchange between two persons.

In summarizing the essential parts of Nash's contribution we will follow mainly the discussion of it by Luce and Raiffa (3, p. 124). On the other hand we will slightly adapt the problem in order to make it comparable to the two person two commodity exchange situation dealt with before.

Suppose two persons I and II have each one commodity, person I has a certain amount of commodity 1, person II an amount of commodity 2. There exists no money to facilitate exchange. A trade takes place if each party agrees to it. By a trade is meant an actual reapportionment of the bundle of goods held by persons I and II. We shall suppose that the utilities associated with each possible trade satisfy the Von Neumann and Morgenstern axioms of utility theory.

Of special interest is the situation where no trade takes place. As no person can force the other person to a situation which is worse than his initial position when no trade has taken place, both persons are assured of a gain in utility by the trade, of at least zero. As we are mainly interested in the gain in utility we may describe the position of no trade at all as the point (0,0) in the 2-dimensional graph of Figure 5 showing the possible gains in utility of person I along the x-axis by u_I and the possible gains in utility of II along the y-axis by u_{II}.

We suppose further that the commodities are completely divisible. Actually Nash does not allow for divisibility. Instead he allows for mixed strategies in the game. The mixed strategies enable him to connect the extreme points in the plane by straight lines. In this way a set of possible utility combinations is formed, which is by assumption compact and convex and contains the origin.

We do not allow for mixed strategies in order to represent the process of bidding as realisticly as possible. But the assumption of infinite divisibility of commodities accomplishes even more than the assumption of mixed strategies. That is, the northeast boundary of the set S of possible combinations of utility gains is convex but may or may not consist of straight line segments. As the worst possible state is indicated by the point (0,0) of no gain in utility, the set of possible combinations of utility gains is the compact and convex set S indicated in Figure 5.

Both persons want to trade such that their utility gain is as much as possible. In other words person I wants to arrive at a point in S as far to the right as possible and person II wants to arrive at a point as high as possible. These wishes are incompatible in general. Still there will be trade according to Nash, as long as there are points in S above and to the right of (0,0).

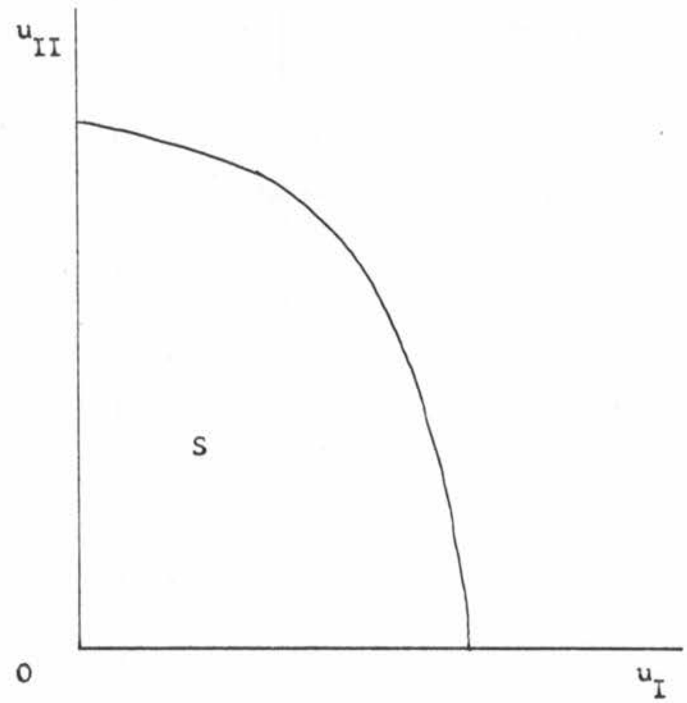


Figure 5. The set S of possible utility gains for persons I and II

The unique solution proposed by Nash is now derived as follows: In the region S find the unique point (u_I^o, u_{II}^o) such that $u_I^o u_{II}^o$ is the maximum of all products $u_I u_{II}$, where (u_I, u_{II}) is in S i.e.,

(1) (u_I^o, u_{II}^o) is a point of S, $u_I^o > 0$, $u_{II}^o > 0$.

(2) $u_I^o u_{II}^o \geqslant u_I u_{II}$ for all (u_I, u_{II}) belonging to S such that $u_I \geqslant 0$ and $u_{II} \geqslant 0$.

The point (u_I^o, u_{II}^o) is called the Nash solution to the bargaining game. The Nash solution is the only point satisfying the following four assumptions (3, p. 126):

(1) Invariance with respect to utility transformations. That is, if the utility functions of I and/or II change origin or are multiplied by a constant, the solution point must change by the same procedure.

(2) Pareto optimality.

(3) Independence of irrelevant alternatives. That is, if the set S of possible utility gains is expanded, the solution shall be either the existing solution point, or shall be contained in the expanded part of S. If the set S is contracted such that the solution point of the original set S still lies in the set, then this point will also be the solution point in the contracted set.

(4) Symmetry. That is, the roles of the players are completely symmetric.

Although we have adapted the bargaining game slightly for our purposes, these adaptions neither implied a change in the assumptions about the compactness and convexity of S, nor a change in the four assumptions mentioned above. Therefore the Nash solution is also the unique solution to the slightly adapted problem.

E. A Comparison between the Competitive Solution and the Nash Solution

In this section we will compare the competitive solution of the two person exchange problem under complete information with the Nash solution to the two person bargaining problem, especially with respect to the utilities of the solution. If there can be no confusion we will often call the utilities belonging to the competitive solution simply the competitive solution.

As we know that the Nash solution is the only point satisfying the four assumptions, given that the set S of possible utility gains is compact and convex, we will investigate successively if these conditions are met by the competitive solution.

Before we actually will do this however, we need to say something about the measuring of utility. One of the reasons for dropping the assumption of a cardinal utility function in general equilibrium theory, was that it was possible to derive meaningful results with a much more general assumption. This assumption is that if a function is a utility function any monotone transformation of it is also a utility function. The competitive solution as far as the commodities is concerned is invariant with respect to monotone transformations and can therefore be derived if only this assumption is used.

However, this does imply that little can be said about the utilities belonging to the solution. Suppose for instance a person has a preference ordering over three commodities A, B and C indicated by $A < C < B$.

Then the person may represent the utility belonging to A by the number u(A) and the utility belonging to B by the number u(B). The only requirement which these numbers have to satisfy is that $u(A) < u(B)$. Given these numbers the only requirement which u(C) has to satisfy is $u(A) < u(C) < u(B)$.

This leaves the utility of the outcome useless as an objective indicator other than of relative utility. At the same time any specific comparison between the competitive solution and the Nash solution can not be made.

Therefore we have to assume that the utilities can be measured in cardinal numbers. More specifically we will assume that for both cases the utility functions of persons I and II are constructed with the help of an utility assignment scheme satisfying the Von Neumann and Morgenstern axioms of utility theory.

As an example of how such a construction would be accomplished let us consider the contract curve of the special symmetric exchange situation of Figure 4. Suppose we call the commodity bundle on the contract curve which is least preferred by person I, A, we call the commodity bundle which is most preferred by him on the contract curve, B and the commodity bundle associated with the competitive solution we call C.

So we have for person I, $A < C < B$. Now suppose with A and B we associate arbitrary utility numbers, but such that $u(A) < u(B)$ as indicated in Figure 6. The assumption underlying the Von Neumann and Morgenstern axioms of utility theory now is, that it is possible to

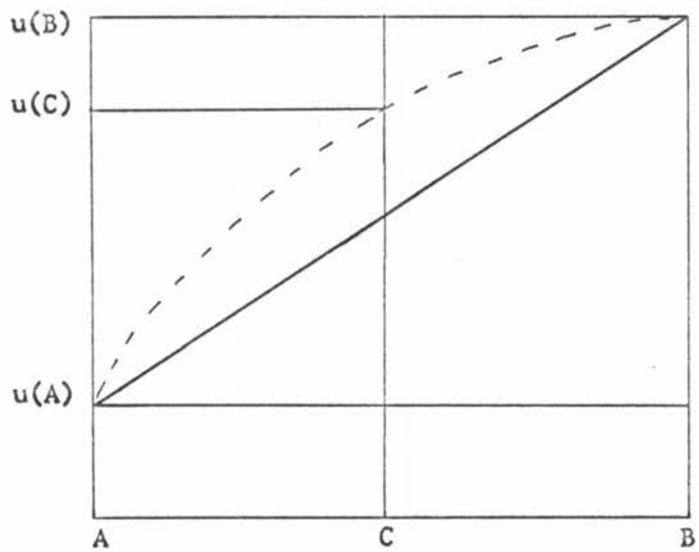


Figure 6. Construction of a utility function for person I

derive the utility of C given the utilities of A and B by a certain procedure involving expectations over random events.

The procedure is as follows: The person, in this case person I, will be asked to say to which probability combination of u(A) and u(B), u(C) is indifferent. The person is supposed to be neutral towards risk taking. Suppose we find the following relation:

$$u(C) = \frac{1}{3}u(A) + \frac{2}{3}u(B)$$

Then we may plot the utility of C in Figure 6 according to this formula. By repeating this process we may find the utilities for person I belonging to all the points on the contract curve. We call this procedure an utility assignment scheme. It is a linear assignment scheme, because it leaves the constructed utility function undetermined with respect to its origin and unit of measurement.

It is important to note that although the assignment scheme determines the constructed utility function up to a linear transformation of it the constructed utility function itself in general will not be linear, as is indicated by the situation in Figure 6.

Now suppose the utility functions of persons I and II are constructed in the prescribed manner. Let us now investigate if the competitive solution satisfies the assumptions of the Nash solution.

Let us therefore transform the region of possible utility gains for both persons in Figure 4, into a set S of possible utility gains for both persons as in Figure 5. In Figure 7 we show this transformation. Figure 7a shows on the x-axis the begin point A of the contract curve of Figure 4, the end point B and the point C of the competitive solution.

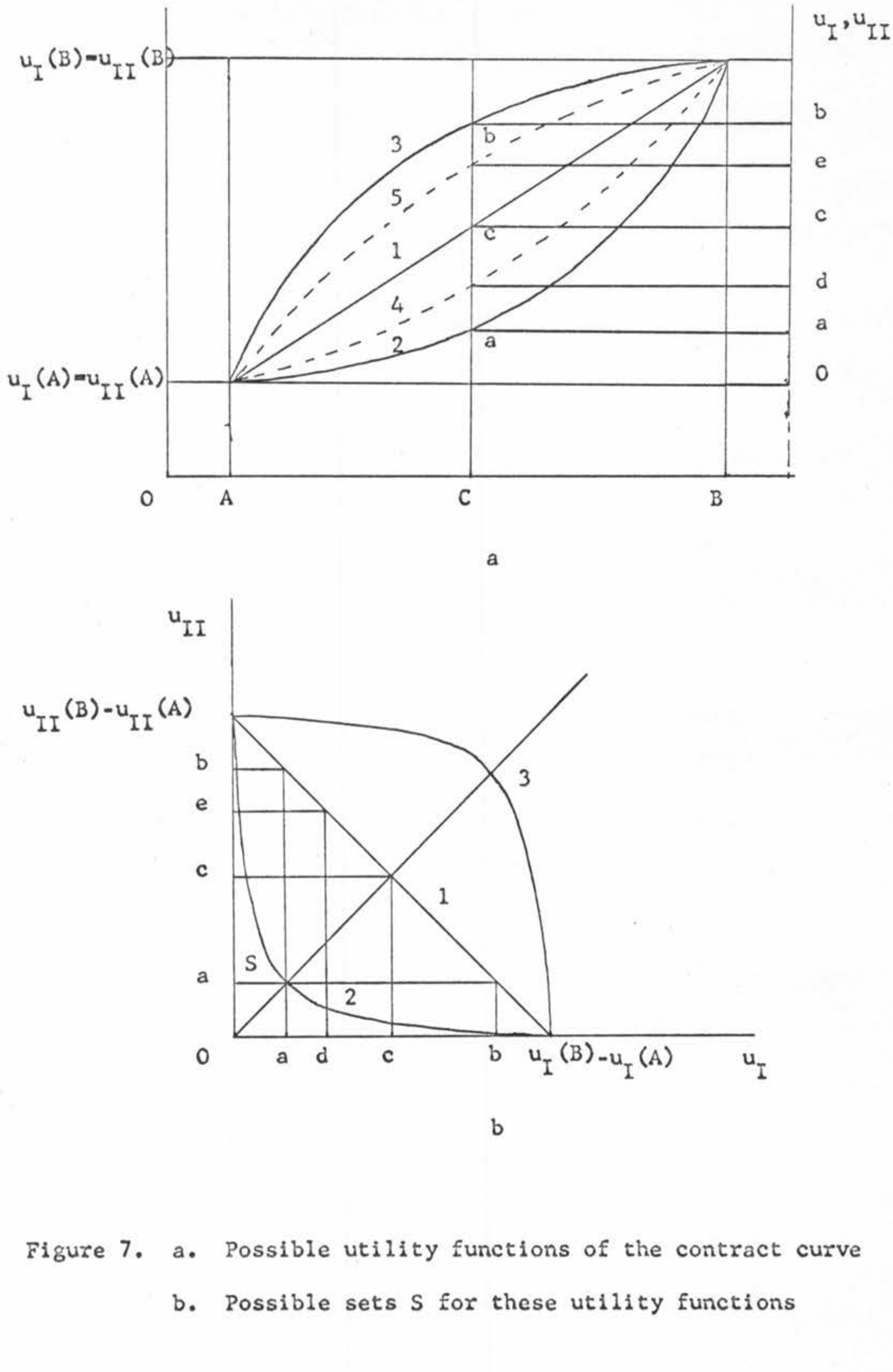


Figure 7. a. Possible utility functions of the contract curve

b. Possible sets S for these utility functions

As the utility functions of persons I and II may be always transformed linearly, we can always make the utilities for persons I and II at the point A the same and at the point B the same for both persons. This we have done in the figure. The utility of the commodity bundle A is indicated by $u_I(A) = u_{II}(A)$. Likewise the utility of B is the same for both persons $u_I(B) = u_{II}(B)$. As we are really only interested in the gains of utility and both persons are always assumed at least of a utility of $u_I(A) = u_{II}(A)$, we may take the utility of A for both persons as the origin in Figure 7b.

So we will only transform Figure 7a into Figure 7b with respect to utility gains. In Figure 7a therefore we have measured the total utilities on the vertical axis to the left of A and the gains in utility on the vertical axis to the right of B. The maximum possible utility gain for person I in the set S of Figure 7b is indicated by $u_I(B) - u_I(A)$ and for II by $u_{II}(B) - u_{II}(A)$.

The procedure for transforming Figure 7a into Figure 7b is now as follows: For any position of person I between A and B in Figure 7a, his utility gain belonging to that position indicated on the right y-axis of Figure 7a, is transformed to a point along the x-axis of Figure 7b, such that the amount of utility gain remains the same. The y-coordinate in Figure 7b is found by taking the position of person II between A and B in Figure 7a corresponding to the position of person I, such that the positions of persons I and II represent a possible exchange situation, and transforming the utility gain for II belonging to that position, to the y-axis of Figure 7b.

Suppose for instance that the constructed utility function for the contract curve is linear for both persons as indicated by the line 1 in Figure 7a. We know from Figure 4 that if person I is at position A, person II must be at position B in order for exchange to be possible. Then the utility gain for person I is 0 and the utility gain of person II is $u_{II}(B) - u_{II}(A)$. So the utility gain of both persons for this situation, is indicated by the point $(0, u_{II}(B) - u_{II}(A))$ in Figure 7b. This is the left end point of the curve 1 in Figure 7b.

If person I moves from A in the direction of B in Figure 7a, person II must move from B in the direction of A in order for exchange to be possible. If person I reaches C, person II reaches it at the same time. Both persons have the same utility at C as is indicated by c on the curve 1 in Figure 7a and by the point (c,c) in Figure 7b.

Although Figure 7a shows only the constructed utility functions for the contract curve, this is really all what we need to construct the set S of possible utility gains for persons I and II in Figure 7b. Because we know that the contract curve is Pareto optimal. This implies that the northeast boundary of the set S in Figure 7b is always the transformed contract curve. Because suppose this is not the case. Then there is a point in S which is not dominated by a point of the transformed contract curve. This implies that there is a point in the region of possible utility gains enclosed by the initial indifference curves and the curves of maximum possible utility gains, in Figure 4, which is not dominated by a point on the contract curve. This is a contradiction.

So if we know the utilities for both persons belonging to points on the contract curve, we can find the northeast boundary of the set S. This implies that we know the whole set S, because the other boundaries are formed by the x-axis and y-axis.

The competitive solution satisfies the condition of Pareto optimality. Therefore it must lie on the northeast boundary of the set S. But although the northeast boundary is Pareto optimal, this does not determine its shape uniquely.

It can be shown that if the utility functions of the contract curve for both persons are linear as indicated by 1 in Figure 7a, the transformed Pareto optimal curve will also be linear as indicated by 1 in Figure 7b. If the utility functions of the contract curve are convex as indicated by 2 in Figure 7a, the northeast boundary of the set S in Figure 7b will be convex with respect to the origin, as also indicated by 2. If the utility functions are concave in Figure 7a as indicated by 3, the northeast boundary will be concave with respect to the origin in Figure 7b, as shown by 3. If the utility function of the contract curve is concave for one person and convex for the other, the northeast boundary of the set S may have several possible forms.

It may be seen now that the utility for each player of the competitive solution, is invariant with respect to linear transformations of any or both of the utility functions of persons I and II. That is, if we multiply the utility gains of any or both players in Figure 7a by a positive constant, the utility gains of any or both persons in Figure 7b change by the same procedure.

We will now check the assumption of independence of irrelevant alternatives. We will show that the assumption is not satisfied by an example. A solution point of the set S of possible utility gains is not independent of irrelevant alternatives, if we can construct a smaller set S, which still contains the solution point of the initial set S, but such that the solution is now another point.

Suppose that the curves 2 and 3 in Figure 7a are such that if we turn line 2 180° in the plane around c, it covers 3 completely. Now suppose the utility curve of both persons I and II is 2 in Figure 7a. Then the utility gain of the competitive solution for both persons is indicated by the point (a,a) of Figure 7b.

But now suppose the utility curve is 2 for person I in Figure 7a and 3 for II. Then by construction the Pareto optimal curve in Figure 7b will be the straight line 1. Moreover the utility gain of the competitive solution for both persons will be the point (a,b) on this line because at C in Figure 7a person I has a utility gain of a and person II of b. In the same way if we change the roles of persons I and II such that the utility curve of person I now is 3 in Figure 7a and of II is 2, then the utility gain of the competitive solution will be the point (b,a) on the line 1 in Figure 7b.

So the point of utility gains for persons I and II belonging to the competitive solution changes for different utility functions of persons I and II. It can be shown, that the point of utility gains in Figure 7b may be any point on the line 1 depending upon the particular form of the utility functions in Figure 7a.

Now suppose the competitive solution is point (a,b) in Figure 7b.

We have to show that (a,b) will not be the solution point of the set S, if we make the set S smaller such that (a,b) is still contained in it. We accomplish this by constructing two utility curves in Figure 7a with the same symmetric properties as the first lines, but both closer to the line 1. These lines we have shown by the dashed curve 4 for person I and 5 for person II in Figure 7a. This still leaves the northeast boundary in Figure 7b as the straight line 1.

So (a,b) will still be on this line. However (a,b) will not anymore be the solution to the problem, as may be seen by finding the utilities belonging to the competitive solution as the intersection of the vertical line at C in Figure 7a with the dashed utility functions. The new competitive solution we have indicated by (d,e) in Figure 7b. So we need only to make the set S smaller to have a result. To accomplish this we may make curve 5 in Figure 7a slightly less concave over a range which will not effect the solution and the position of point (a,b) in Figure 7b.

So the result is that the point of utility gains for persons I and II belonging to the competitive solution is not independent of irrelevant alternatives.

Finally it may be seen that the competitive solution does also not satisfy the assumption of symmetry. The set S in Figure 7b with northeast boundary 1, is completely symmetric. This would imply that the solution would be the point (c,c) in order to satisfy the assumption of symmetry. We have already seen that the solution may be any point on this line.

So we have found that the point of utility gains belonging to the competitive solution satisfies only two of the four assumptions of the Nash solution, namely the assumption of invariance with respect to linear transformations and the assumption of Pareto optimality.

There only remains to be investigated the condition of compactness and convexity of the set S of possible utility gains. A set is said to be compact if it is closed and bounded. The finite numbers assigned to the utilities of the contract curve imply that the boundedness condition is satisfied. The Pareto optimality of the contract curve implies that the boundary is included in the set S. Furthermore the boundaries of the set S consisting of the axes are also included in the set.

As the discussion of Figure 7 already showed, the set S of possible utility gains for persons I and II, belonging to the bilateral exchange case, need not be convex. Especially when the utility functions of both players are convex the northeast boundary will also be convex. Therefore the competitive solution will not satisfy in general the condition that the set S of possible utility gains is convex.

III. A SPECIAL HYPOTHESIS

A. Introduction

In this chapter we will first state the conditions under which we want to consider the isolated exchange situation. Next we will investigate the usefulness of the competitive solution and the Nash solution under these conditions. Finally we suggest and criticize a special solution based upon the criticism of the competitive solution and the Nash solution.

B. Special Assumptions

In the first place we want to restrict ourselves to the case of isolated exchange under complete information. This certainly would be a severe limitation if we were mainly concerned about the solution of the isolated exchange situation as such. For the case of isolated exchange may be considered as a peculiar balanced situation of which the solution will be determined mainly by factors not known beforehand. The parties may set out to find the determining factors of the solution by a bargaining process which is at the same time a learning process about the determining factors the other party may have and a hiding process of the determining factors the person himself may have. In short it is often said that the solution of the isolated exchange situation will be determined in a great deal or in part by the particular bargaining skills of the parties.

We will not consider these specific determining factors. That is, we will exclude the effect of bargaining skills upon the solution. If we exclude these effects under complete information, it may be possible to say something about the influence of specific other factors. In particular we want to take into consideration the effect on the solution of interpersonal comparison of utility. To be able to do that therefore, we have to assume that interpersonal comparison of utility is possible at all.

More explicitly, we will assume that the utilities of persons are representable through a linear utility assignment scheme satisfying the Von Neumann and Morgenstern axioms of utility theory and such that the utilities of all persons are measured with respect to the same origin and are multiplied by the same constant.

We will assume a barter economy in which no explicit prices exist but only exchange ratios. Any party makes bids in the market specifying the amount of each commodity he offers in exchange for a specific amount of another or other commodities. For simplicity demand may be considered as a negative offer. Of course these exchange ratios may be considered as accounting prices.

Furthermore we want to consider the exchange process as a specific game. In order to be able to consider the situation as a game at all, certain conditions have to be satisfied. In general a game in normal form consists of (3, p. 55):

(1) The set of n players

(2) n sets of pure strategies, one set for each player

(3) n linear payoff functions, one for each player, whose values

depend upon the strategies of all the players.

The linear payoff functions we have called before linear utility assignment schemes. They have to satisfy the assumption of interpersonal comparison of utility. The players are also assumed to have full knowledge of all the strategy sets and all the payoff functions of the players. They are also rational, that is they will always choose the alternatives with largest utilities.

Of all the possible games we will choose further the class of games, in which no preplay communication is possible. This implies at the same time that we will not allow for binding agreements. The class of games in which no preplay communication is allowed in order to form binding agreements, is called the class of non-cooperative games (3, p. 89).

As the theory of games as developed by Von Neumann and Morgenstern (8), allows for preplay communication, we can not apply the results of this theory to our cases. It is exactly the incorporation of all possible results of preplay communication in the solution, which prevents them from deriving solutions in terms of unique points for general n-person games.

This is certainly unsatisfactory as far as the economic interpretation of the theory is concerned. Many attempts have been made to derive more specific results by still using the method of game theory. Nash (5) attacked the problem of preplay communication by considering it also in terms of game theory. In excluding preplay communication we suppose to work in the same spirit. It seems to us that the results of the game theoretic approach should not depend on considerations not dealt with explicitly within the body of the theory. In terms of game theory we could say that the result of the game under consideration should not depend on previous games of which the results are not known.

But the exclusion of preplay communication as such does not guarantee a solution which is more determinate. In order to arrive at more specific solutions therefore we have to look for other possibilities allowable within the game theory set up.

As we want to use the tools of game theory in order to investigate exchange behavior, a method which presents itself naturally is the possibility of repeated playing of a game. The repetition of the plays may be compared to the repeated bidding as dealt with in the previous chapter. At any play we will only allow for single strategies. A more detailed description of the repeated play of the game will be given later on.

Through the process of repeated playing of a game communication is established between the players. This communication may or may not lead to agreements, which are only binding if the game is terminated immediately after agreement is reached. The rules of the game therefore determine the possibility of binding agreements. The repetition of the plays allows for the possibility of cooperation between players. At the same time it allows us to investigate if there is reason to expect certain specific types of cooperation. If this is the case, it will enable us to be more specific also about the solution of the particular game.

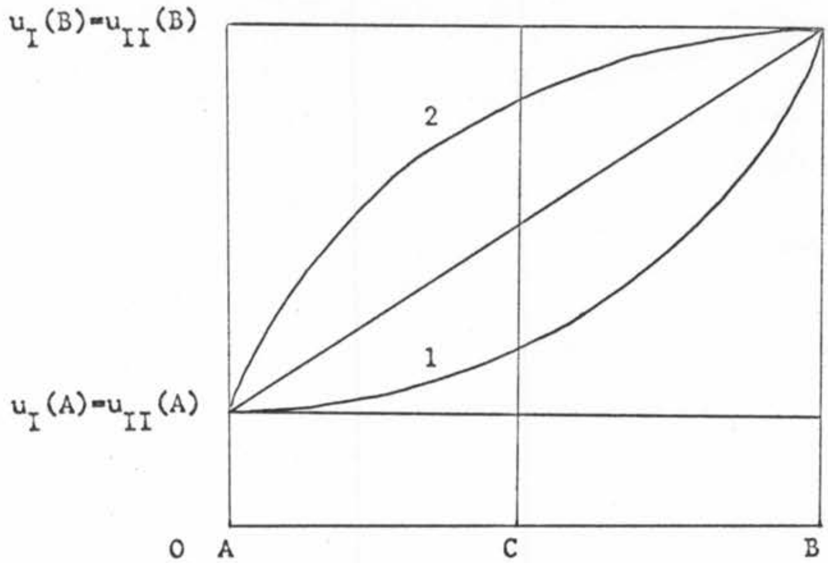
C. A Criticism of the Competitive Solution and the Nash Solution

In this section we will discuss the reasonableness of the competitive solution and the Nash solution as a solution of the special type of isolated exchange situation we want to consider.

In general the difference between the Nash solution and the competitive solution seems to be that the Nash solution is based upon the principle, that at any moment the division of utilities must be as equal as possible, while the competitive solution is based upon the principle, that the person who will increase his utility most from the acquiring of a certain commodity will also indeed increase his utility most.

Figure 8 demonstrates this. Figure 8a shows two possible utility curves for each person of the contract curve of Figure 4. Figure 8b shows the boundary of the set S of possible utility gains, derived from these curves. If 1 in Figure 8a is the utility curve of person I and 2 of person II, then the competitive solution is C_I in Figure 8b. If 1 is the utility curve of person II and 2 of person I in Figure 8a, then C_{II} is the competitive solution in Figure 8b. The Nash solution is indicated by N in Figure 8b.

At the beginning of the exchange both parties have a utility of $u_I(A) = u_{II}(A)$. We see that the competitive solution increases most the utility of the person with steepest utility hill at A. On the other hand the Nash solution disregards any difference in utility hills and divides the utilities equally among the persons. We may also say that the Nash solution tends to equalize utilities, while the competitive solution tends to equalize commodities.



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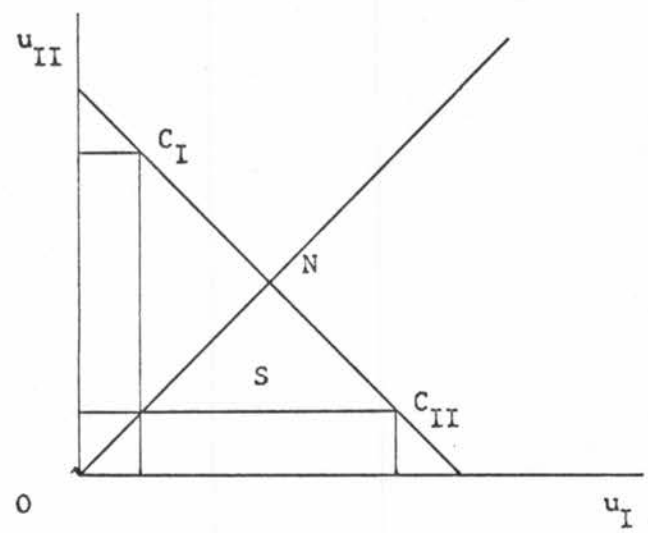


b

Figure 8. a. Possible utility functions of the contract curve

b. Possible solutions in the set S

Disregarding for the moment the reasonableness of each of the solutions per se and the effect of interpersonal comparison of utilities, it seems to us that the likelihood of one of these solutions will depend upon the circumstances under which the exchange takes place.

Suppose for instance that the exchange situation represented in part by Figure 8a happens only once under complete certainty. Then, as both persons need to cooperate for a solution, it seems reasonable that the solution will be close to the Nash solution. Actually there is some experimental support for this reasoning. In "Bargaining and Group Decision Making" (7), Siegel and Fouraker reported the experiments they conducted in order to test several hypotheses about solutions of the bilateral monopoly case under equal bargaining strength.

A linear model was developed in which the parties in the bilateral monopoly case were a single buyer and a single seller of one commodity. From the linear model payoffs were derived for quantity-price combinations. The payoffs were in money. Furthermore the set S of possible utility gains, if constructed, would be symmetric with the Pareto optimal curve forming the northeast boundary being a straight line. The tests were performed with students in the roles of buyer and seller. Each student received the information necessary to perform the specific experiment.

One person was randomly assigned to make the first bid to the other person. After receiving the bid the other person could accept the bid or make another bid. The bidding was allowed to continue for a specific amount of time. The parties had to try to reach agreement within that time in order to receive the money payoffs belonging to the utilities of the solution.

We could derive from Figure 4 a similar payoff matrix as used by Siegel and Fouraker. Of particular interest for us are the results of the experiments conducted under complete information of both buyer and seller (7, p. 58). Of the eight couples which conducted the experiment, six divided the joint payoff in half and the other two almost in half. The six which divided the joint payoff in half, also reached a Pareto optimal point, the other couples almost. These results therefore may be interpreted as supporting the Nash solution for symmetric exchange situations which occur only once.

On the other hand, if the exchange is repeated several times with the utility curves of persons I and II changing over time and if furthermore the information is incomplete such that each party knows only his own utility curve, it may well be that the costs involved in reaching an equal division of the payoff each time are more than offset by the average gain in utility if the competitive solution is accepted as solution each time.

As the experiments conducted by Siegel and Fouraker (7, p. 70) show, even if the exchange situation occurs only once under incomplete information, the results deviated already more from an equal split solution, although they did not tend more to the competitive solution. As they did not conduct repeated experiments by the same persons, we can not support our reasoning further by experimental evidence from their experiments.

However as we will deal only with exchange situations which occur only once, we may concentrate on the Nash solution as suggested by the results of the experiments. In order to investigate the reasonableness of the Nash solution for the particular exchange situations we want to consider, let us look again at the four assumptions which the Nash solution has to satisfy. Furthermore let us consider also again the conditions of compactness and convexity of the set S. For a detailed discussion of the criticisms of the four assumptions which the Nash solution has to satisfy, we refer to Luce and Raiffa (3, p. 128). We will summarize only the objections relevant to our discussion.

For convenience we will start with assumption 4 and work back to assumption 1. Assumption 4 requires symmetry. It seems to us that this is a reasonable assumption. A problem arises however when one person acts on his own behalf and the other person acts on behalf of a group of persons. We will exclude this possibility.

The assumption of independence of irrelevant alternatives has been the object of severe criticism. The criticism mostly takes the form of an example of comparisons of two extremely different sets of utility gains, but such that the solution point is still the same according to the assumption.

We do not feel that this criticism is relevant to the Nash solution. It seems to us that most of the criticism really is directed towards the assumption of no interpersonal comparison of utility with which assumption we will deal later on. Although we do not agree therefore with the criticisms against the assumption of independence of irrelevant alternatives, we still will not require this assumption for our specific hypothesis. The reasons we will explain later on.

The assumption of Pareto optimality we will discuss in combination with the assumption of convexity of the set S. We have seen already before that the set S of possible utility gains, derived from the region between the indifference curves belonging to the initial positions of the persons in Figure 4, is not convex in general.

Therefore we have to drop the convexity assumption if we want to deal with more general cases of bilateral exchange. But we may replace the convexity assumption in part by an assumption which naturally presents itself, namely Pareto optimality. We have seen already that if the set S represents the possible utility gains in the bilateral exchange case of Figure 4, the northeast boundary must be the transformed contract curve. This curve is Pareto optimal.

But the exchange situation dealt with in Figure 4 is rather special as we have seen. If we assume Pareto optimality of the northeast boundary of the set S, we have to show therefore which exchange situations in general do show a Pareto optimal boundary of the set S. We will show later on as we deal with two person and three person exchange situations, under which conditions the northeast boundary of the set S will certainly be Pareto optimal. At this particular point it may be seen, that the northeast boundary will not be Pareto optimal, if the utility function of one person is decreasing and the utility function of the other person is increasing over the same domain. The case of inferior goods for one person could exhibit the property of a decreasing utility function.

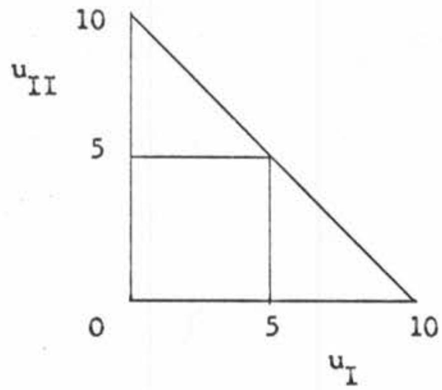
So we assume that the set S is compact and has boundaries consisting of the non-negative axes and a Pareto optimal curve between these axes.

Assumption 1 states that the solution point must be invariant with respect to linear utility transformations. This assumption implicitly assumes that interpersonal comparisons of utility can not be made. That is, if the utility function of person I remains the same and the utility function of person II is multiplied by a constant and has added a constant to it, the solution remains the same for person I. We know that the competitive solution can also be derived without the assumption of interpersonal comparison of utility.

As we have indicated already, we want to consider isolated exchange situations if interpersonal comparison of utility is possible. What will be the effect on the Nash solution if interpersonal comparison of utility is possible? Let us again follow the discussion by Luce and Raiffa (3, p. 130).

In Figure 9, we have shown two different sets S of possible utility gains for persons I and II. In Figure 9a the situation is completely symmetric and the Nash solution is the point (5,5). In the case of Figure 9b the situation is asymmetric and the Nash solution is the point (5,50).

But if interpersonal comparison of utility is possible, is (5,50) then a "fair" solution? That is, is it reasonable to expect that person I will cooperate with person II to get this solution? Person I may assume that the point (9,9) of equal division of the utility gains is a fair division on the basis of the arguments that both persons started at the point (0,0) and that there is no reason to decide upon an asymmetric



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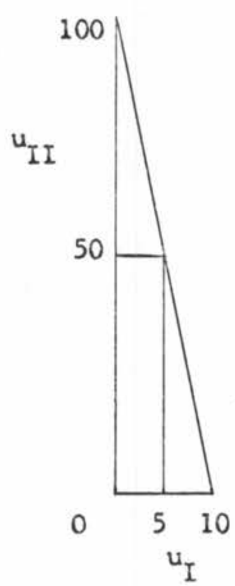

b

Figure 9. a. A symmetric set S for persons I and II

b. An asymmetric set S for persons I and II solution if the initial positions are symmetric. Person I may support his arguments by threatening not to cooperate for any solution different from (9,9). He may gain nothing if no agreement is reached, but then person II will also gain nothing.

On the other hand person II may argue that any solution different from (5,50) is unfair, because only at (5,50) each person gains half of his maximum possible utility gain. Person I would again argue that for him the maximum possible utility gains are not the basic reference points, but the initial positions and therefore .....

This type of argument certainly does not suggest a unique solution to the isolated exchange situation. It does suggest however that the solution will be within a certain range with endpoint on the one side the solution which divides the possible utility gains in half and on the other side the Nash solution.

D. A Special Hypothesis

In the previous section we argued that the Nash solution would be a more likely solution of the isolated exchange situation, if no interpersonal comparison of utility was allowed, than the competitive solution. However the Nash solution is based upon certain assumptions which we want to relax. The basic changes we want to make are that the set S of possible utility gains need not be convex. However the northeast boundary should be Pareto optimal. Furthermore we want to consider interpersonal comparison of utility possible. We will still require that the solution satisfies the assumptions of Pareto optimality and symmetry.

Let us consider therefore a set S of possible utility gains, with northeast boundary not concave to the origin but Pareto optimal and further with different maximum possible utility gains for persons I and II. The set S may be considered representing a two person two commodity exchange situation.

Figure 10 shows such a set S. The northeast boundary of the set is the line AEHNB. We have drawn two straight lines, AB and AC. AB is the straight line connecting the endpoints of the Pareto optimal curve. The straight line AC connects the endpoint A with a point on the y-axis at equal distance from the origin as A.

Consider first the triangle OAB. If the persons would divide the possible utility gains in half, the solution would be P. P is the intersection of the line through (A,A) with the Pareto optimal curve. If the persons would decide to maximize the product of their utility gains, that is if they decided that the Nash solution would be the solution, the solution point would be R. R is the intersection of the line through (A,B) with the Pareto optimal curve.

We could say that the point R is the solution if there is no interpersonal comparison of utility at all. Any change in the utility of one person does not affect the other person as far as the solution is concerned. On the other hand the point P could be considered the solution if there was an extreme effect of interpersonal comparison of utility, such that any change in utility of one person would affect the other person as well as far as the solution is concerned.

The hypothesis is now, that for the special case of the set S being

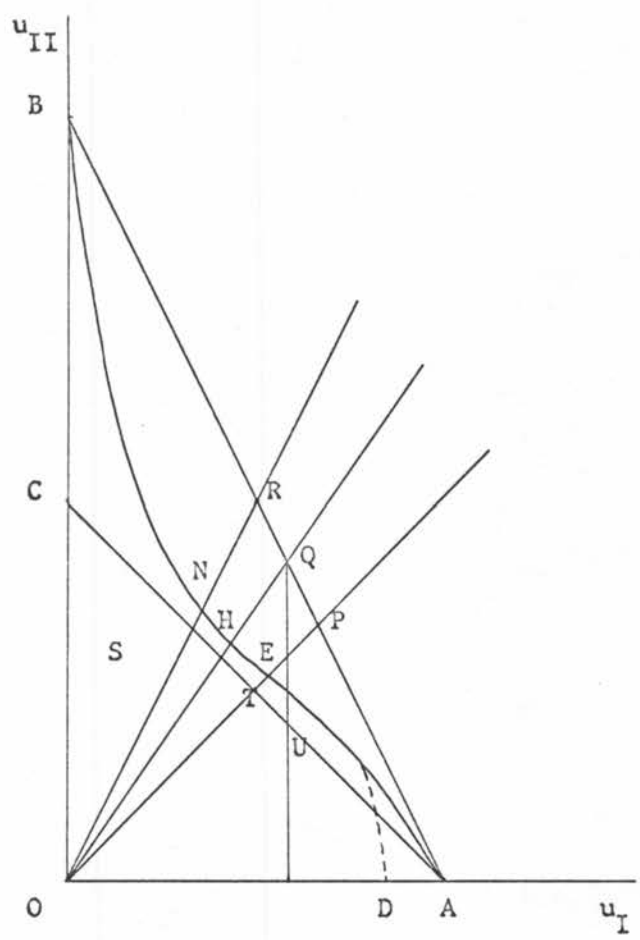

Figure 10. Construction of the solution in the set S

the triangle OAB, the solution point will be the line through the point:

$$\alpha P + (1 - \alpha)R \qquad 0 \leqslant \alpha \leqslant 1 \qquad 4$$

That is, we assume that the effect of the interpersonal comparison of utility will be that the solution will lie somewhere between the Nash solution and the solution where both persons divide the utility gains in half. In the case that B = A, the points P and R will be the same and the solution will be a division of the utility gains in half.

We have used the restriction that the set S is the triangle OAB. That is, the set S was still convex. Now let us consider the set S with northeast boundary AEHNB which is not convex. The line through (A,A) intersects the Pareto optimal boundary at E and the line through (A,B) at N. N however can not be anymore considered as the Nash solution. This implies that we can not in general use the Nash solution as one endpoint of the range of possible solutions.

However another candidate for endpoint suggests itself as the intersection point of the line through (A,B) with the Pareto optimal curve i.e., the point N. The hypothesis is therefore that the solution of the exchange situation represented by the set S of possible utility gains with northeast boundary AEHNB will be the intersection of the Pareto optimal curve with the line through the point:

$$\alpha E + (1 - \alpha)N \qquad 0 \leqslant \alpha \leqslant 1 \qquad 5$$

In general if we call the endpoints of the Pareto optimal curve of any set S, A and B, and if we call the intersection of the line through (A,A) with the Pareto optimal curve E and the intersection of the line through (A,B) with the Pareto optimal curve N, then the solution according to the hypothesis will be the intersection of the Pareto optimal curve with the line through the point:

$$\alpha E + (1 - \alpha)N \qquad 0 \leq \alpha \leq 1 \qquad 6$$

Basically this hypothesis considers only the endpoints of the Pareto optimal curve as determining factors of the solution. More specifically the hypothesis neglects the effects of the particular shapes of the Pareto optimal curves on the solution.

We might consider the point E as the best possible solution for one person and the point N as the worst possible solution. Then formula 6 might be considered as a weighted average between the best and worst possible solution for a person. This would remind of the Hurwicz pessimism-optimism index criterion for decision making under uncertainty (3, p. 282). The resemblance is only superficial however, as the Hurwicz criterion is applied to each strategy of a person in order to determine his optimal strategy, while the formula 6 determines a solution for one particular exchange situation.

If $\alpha = 1$ in formula 6, the persons will divide the possible utility gains in half at the Pareto optimal curve. If $\alpha = 0$, they will divide it on the Pareto optimal curve determined by the ratio of their maximum possible utility gains. If the Pareto optimal curve is concave this implies that they will divide the possible utility gains close to or equal with the Nash solution.

The exact value of α may be determined by experiment rather than by reasoning. For convenience however we will assume that $\alpha = \frac{1}{2}$. We have shown the solution for $\alpha = \frac{1}{2}$ in Figure 10 as H on the Pareto optimal curve.

E. A Criticism of the Hypothesis

In this section we will investigate some of the limitations of the hypothesis. First of all, it will be shown that the assumption of invariance with respect to linear utility transformations and the assumption of independence of irrelevant alternatives are not satisfied.

Suppose the set S of possible utility gains for persons I and II is the triangle OAB in Figure 10. The utilities of the solution of the bilateral exchange situation are indicated by the point Q according to the formula 6. Now suppose we multiply the utilities of person II such that B becomes the point C. Then the point Q is transformed in the point U. In order therefore that the assumption of invariance with respect to linear utility transformation is satisfied, the solution must be U. However applying the formula 6 to the new situation the solution will be the midpoint T between A and C. As T $\neq$ C the assumption is not satisfied.

To investigate the assumption of independence of irrelevant alternatives let us slightly change the Pareto optimal curve AEHNB of the set S in Figure 10, such that we get the Pareto optimal curve DEHNB. The new set S is smaller than the initial set S and still contains the solution point H of the initial set. According to the assumption, H must also be the solution of the new set S. As the point D is to the left of A however, the point N on the Pareto optimal curve will move to the left. As the point E remains the same, the point H must also move to the left. Therefore this assumption is also not satisfied.

Although these specific assumptions are not satisfied, this need not to argue against the hypothesis. We would rather suggest that it argues against the assumptions, if the hypothesis may be considered as a reasonable explanation of behavior under the specific circumstances.

Finally we see that the assumptions of symmetry and Pareto optimality are satisfied.

A more serious criticism might be directed towards the fact that only the endpoints of the Pareto optimal curve have effect on the solution through their position in the formula 6. This implies that if the maximum possible utility gains of both persons are equal, they will divide their utility gains equally, whatever may be the form of the Pareto optimal curve.

In Figure 11 we have shown three possible Pareto optimal northeast boundaries of the set S, which is such that the maximum possible utility gains of persons I and II are equal. Let us first consider the curve 1. The solution point according to the formula 6 is the point (a,a). Now it may seem reasonable that person I would be able to get a little bit more than the amount a from the exchange because he can increase his utility gain greatly without reducing the utility gain of II very much.

The formula neglects the possibilities surrounding the proposed solution, by assuming that person II will stick to the idea that both persons have equal maximum possible utility gains and therefore will not cooperate for any solution other than an equal division of the utility gains.

Of course it makes a great difference if the exchange situation occurs only once, or occurs repeatedly over time. The hypothesis only

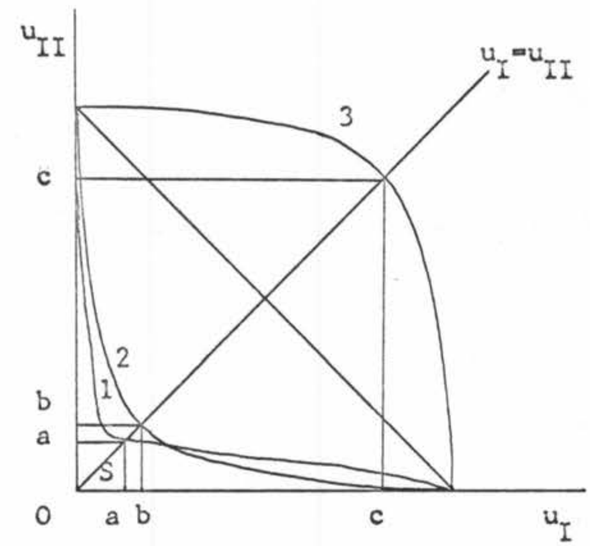

Figure 11. Three possible northeast boundaries of the set S

deals with the case of exchange taking place only once. Because as soon as we suppose that the exchange situation occurs an indefinite number of times, each person can benefit greatly by cooperating such that the joint utilities are maximized over time. In the case of the convex curve 1, this would imply that person I would receive all the utility gain at one exchange situation and II at the next and so on. Because, given the particular form of the curve 1, no combination of two solutions will yield an equal or greater total utility gain and at the same time divide it in half between persons I and II.

Now let us consider the northeast boundaries 2 and 3 together. The curves 2 and 3 are symmetric around the line $u_I = u_{II}$. The solution points are (b,b) and (c,c) respectively. The point (c,c) seems a very reasonable solution point for the particular exchange situation, because not only the roles of persons I and II are completely symmetric but also because the sum of the utility gains is maximized at this point. That is, even if the exchange situation is repeated (c,c) seems to be reasonable as a solution.

At the point (b,b) on curve 2 however the sum of the utility gains of the two persons is not maximized as a movement away from (b,b) will increase the utility of one person more than it will decrease the utility of the other person. Each person may think that he has a reasonable chance of success if he tries to get a little bit more than the amount b. That is, if the curves are convex such as the curve 2, the respective bargaining skills of the persons may play an increasingly more important role in determining the solution of the exchange situation. As long as nothing is known about the particular bargaining skills of the persons, nothing can be said about its effect on the solution. Therefore we have excluded the possibility of different bargaining skills. In testing the hypothesis therefore the effects of bargaining skills have to be excluded. In that case the hypothesis is that the point (b,b) will be the solution. It is to be expected however that the deviation of the solution from (b,b) will be greater than the deviation from (c,c).

Finally it may be argued that the solution proposed by the hypothesis is not uniquely determined by the assumptions as is the Nash solution. Given the special assumptions under which we consider isolated exchange, this objection does not seem to be a serious criticism. We want to deal with isolated exchange under complete information. However we are not primarily concerned with the solution of the exchange problem as such. We are concerned with the effects on the solution if certain elements which have influence on the solution are changed. We may develop a set of axioms and then show that the solution logically follows from the axioms, but with the danger of destruction of the whole theory if the solution is rejected by experiment.

Or we may develop a more general set of axioms, which will certainly be satisfied by the solution, such that the solution is not the only possible solution based upon the axioms. A rejection by experiment may then cause an adaption in the solution rather than in the axioms. We have used the last more statistically oriented approach.

IV. THE TWO PERSON CASE

A. Introduction

In the previous chapter we have discussed a special hypothesis about the solution of a two person exchange situation represented by a set S of possible utility gains. The hypothesis is expected to be true if certain conditions are satisfied. In this chapter we shall consider more specificly the conditions. In particular we will discuss how an isolated exchange situation may be represented by a game. We will derive a payoff matrix for the game and discuss the rules of the game. From the payoff matrix we may derive again the set S of possible utility gains.

We will start with the particular exchange situation shown by Figure 4. This figure represents a special symmetric exchange situation which will have a symmetric solution, according to the hypothesis. Afterwards we will generalize the exchange situation by introducing asymmetry in the utility functions and in the initial positions of the persons. We will adapt the hypothesis if necessary until finally it will be able to deal with all possible two person m-commodity situations.

First of all however we have to be sure that we deal only with exchange situations which are such that the northeast boundary of the set S is Pareto optimal over its whole range. This will be guaranteed if the constructed utility functions of both persons are continuous and strictly increasing in the same direction, say to the right. If the utility function of a person is continuous but not strictly increasing or decreasing, it will not be possible to construct a contract curve with increasing or decreasing utilities for that person. This implies at least that the maximum possible utility gain of the person remains constant, while the maximum possible utility gain of the other person changes. Over this range therefore the northeast boundary of the set S will not be Pareto optimal.

If the utility curve of one person is decreasing and of the other person increasing, then the contract curves will be such that the utility for both persons increases, if they move in opposite direction along their contract curves. This implies also that the northeast boundary can not be Pareto optimal over its whole range.

If the utility functions of both persons are strictly increasing in the same direction however, the contract curves will be strictly increasing also in the same direction. At any point on his contract curve a movement of one person along his contract curve must be offset by a movement of the other person along his contract curve in opposite direction in order for exchange to be possible. Thus any increase in utility along the contract curve of one person will be accompanied by a decrease in utility of the other person along his contract curve. This implies Pareto optimality. As the contract curves are continuous, the increase and decrease in utility along it are continuous. Therefore the northeast boundary of the set S is continuous and Pareto optimal.

B. The Play of the Game

We will now construct a payoff matrix which may be considered representable for the exchange situation of Figure 4. We recall that persons I and II have the same indifference curves which are orthogonal hyperbolas. The initial positions of persons I and II are on the same indifference curve and symmetric with respect to the line k, which has an angle of 45° with the x-axis.

As a strategy we will define any amount of a commodity offered for exchange for any other amount. From the infinite number of strategies possible by this definition, we will only consider strategies which represent reasonable chances of success. With reasonable we mean strategies which do not lead to losses for any party. As any party can always choose not to trade, this seems to us no serious restriction.

So we will consider only strategies such that the new positions of the persons will be in the closed region IAIIB of Figure 4. This still leaves an infinite number of strategies for both persons. Apparently we have to restrict the possible strategies still further. We suspect the solution to be at least Pareto optimal. Let us therefore consider a set of strategies which lead to a Pareto optimal solution and still a few other strategies which do not lead to a Pareto optimal solution but may be candidates for some reason for a different solution, such that all the essential strategic possibilities of the game may be considered as represented by the chosen strategies.

In Figure 12 we have preserved the necessary parts of Figure 4. Furthermore we have indicated by numbers the possible strategies of a person. For instance the number 1 means that person I or II makes a bid such that, if the bid is accepted, he will be in position 1 after the exchange. We have shown 13 different strategies.

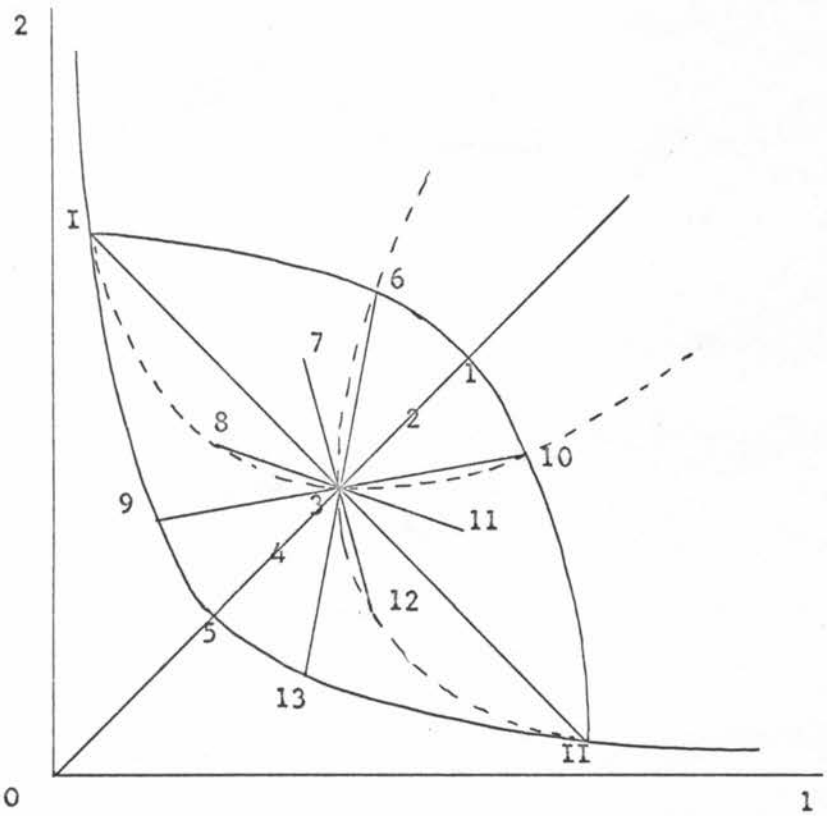

Figure 12. Thirteen representable strategies

In Figure 13 we have shown the payoff matrix for these strategies. The payoff matrix shows only the utility gains of the persons. The first number in a payoff square is the utility gain of person I and the second number the utility gain of person II. Empty squares indicate that the utility gains for both persons are zero. Only one square in each row or column has non-zero entries. This is due to the fact that for any strategy of person I there is only one strategy of II, such that exchange actually can take place. At all the other strategies exchange will not take place and the payoff will be zero for both parties. For our particular example we have chosen the utilities such as to preserve the situation of Figure 4 as realistically as possible. We have attributed a maximum possible utility gain of 10 to both parties.

The upper left diagonal shows the utility gains belonging to the solutions on the contract curve. From the payoff matrix we may again construct the set S of possible utility gains for both persons. In Figure 14 we have shown the payoffs as dots in the set S of possible utility gains. To find the northeast boundary we may connect these dots by straight lines such that no dot is excluded from the set S. A more exact approximation of the set S we get if we transform the utilities of Figure 4 directly to the set S of Figure 14.

We may find the solution for this particular exchange situation by applying formula 6 of page 51. As the maximum possible utility gains are equal for both persons, the points E and N in the formula will be the same and the solution will be the intersection of the Pareto optimal curve with the line through (10,10). The solution will be the point (4,4).

II

I

	1	2	3	4	5	6	7	8	9	10	11	12	13
1					10,0								
2				7,2									
3			4,4										
4		2,7											
5	0,10												
6													9,0
7												4,3	
8											3,4		
9										0,9			
10									9,0				
11								4,3					
12							3,4						
13						0,9							

Figure 13. The payoff matrix

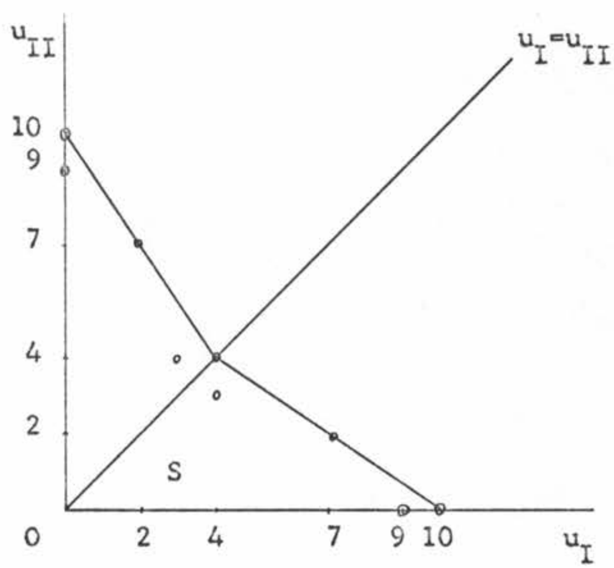

Figure 14. The set S of possible utility gains

The parties will divide their possible utility gains in half.

The payoff (4,4) will be the solution of a particular two person non-zero sum game of which the characteristics are the following: No preplay communication is possible. The only communication between the players takes place through the announcing of their strategies. Both players are completely informed about the payoff matrix of both persons.

Player I starts the playing of the game by announcing a strategy to player II. Then player II announces a strategy to player I. The game is ended if the strategies announced by both players have a corresponding payoff square with at least one positive entry. If this is not the case, player I announces a strategy for the second time, which ends the game if it results, together with the strategy of II, in a payoff with at least one positive entry. If not, II announces a strategy and so on, until a payoff with at least one positive entry is reached, or until a certain amount of time has passed, which ends the game. If the payoff has at least one positive entry at the end of the game, the actual exchange suggested by the strategies, takes place and the parties receive the payoff.

To test the hypothesis the payoffs would be in money. The effect of different utilities of money and of bargaining skills would then be excluded by proper randomization procedures.

As we have indicated already before, considering the payoff matrix of Figure 13, we see that the upper left diagonal of non-zero squares represents the Pareto optimal curve of the set S. This implies that for any person any strategy with a number higher than 6 is dominated by at least one strategy with a number lower than 6. This suggests strongly that both persons will mainly if not only announce strategies with numbers lower than 6.

Suppose for instance that person I announces the strategy with number 11. Then II can announce strategy 3 such that person II increases his utility gain without person I changing his utility gain. The same is true if the roles of persons I and II are interchanged.

As was pointed out already in the previous chapter, the experiments conducted by Siegel and Fouraker support the hypothesis, that the parties will divide their utility gains in half on the Pareto optimal curve. In this special symmetric situation the solution point (4,4) is also the competitive solution.

C. Unequal Maximum Possible Utility Gains

We will now gradually increase the asymmetry between persons I and II in order to be able to deal with all possible two person isolated exchange situations. First of all let us suppose that the utility of the point A on the contract curve in Figure 4 is the same for both persons, but that the utility of the point B is two times as high for person II as for person I. The effect of this difference on the set S of possible utility gains will be that the utility gains of person II are multiplied by 2. Figure 15 shows the resulting set S.

We may find the solution for this particular exchange situation by using formula 6 of page 51 and furthermore by assuming that $= \frac{1}{2}$. The line through the point (A,A) corresponds now with the line through (10,10).

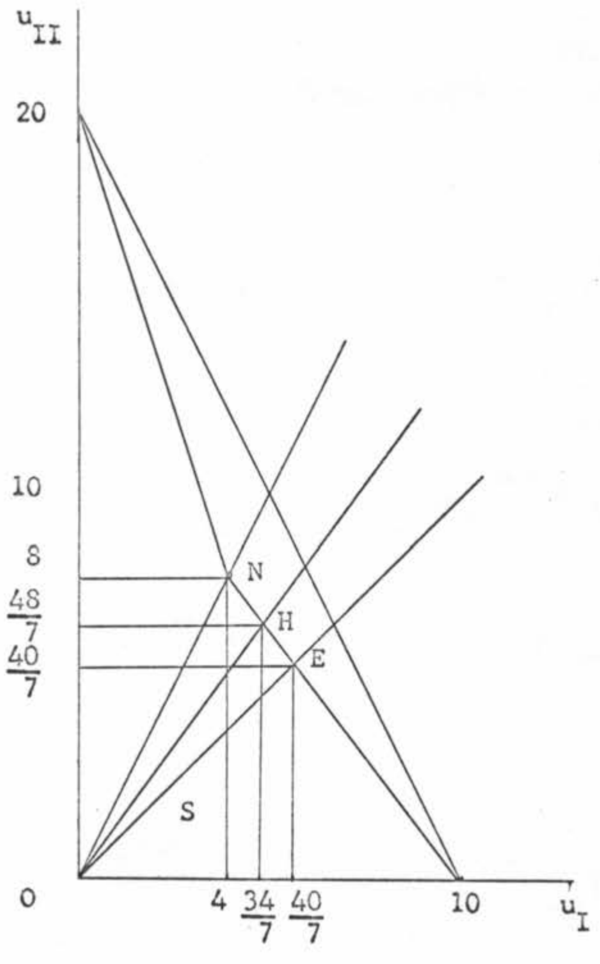

Figure 15. The solution in the asymmetric set S

A little calculation will show, that the intersection of this line with the Pareto optimal curve will be the point $(\frac{40}{7},\frac{40}{7})$. The line through the point (A,B) corresponds with the line through the point (10,20). The intersection with the Pareto optimal curve is the point (4,8). The solution will now be the intersection of the Pareto optimal curve with the line through the point $\frac{1}{2}(\frac{40}{7},\frac{40}{7}) + \frac{1}{2}(4,8) = (\frac{34}{7},\frac{48}{7})$, or about (5,7). As the Pareto optimal curve between $(\frac{40}{7},\frac{40}{7})$ and (4,8) is a straight line in our particular example, the point $(\frac{34}{7},\frac{48}{7})$ is the solution. We have indicated the solution by the point H in Figure 15.

The hypothesis may be tested again by the procedure of repeated play of a game of which the payoff matrix is the same as the one shown in Figure 13, except that the entries for person II are multiplied by 2. Of course we could have multiplied the utility gains of person II by a number greater than 2. This may be necessary if we want to get significant results with as few experiments as possible. Furthermore the solution (5,7) is not a payoff in Figure 13 if the entries for person II are multiplied by 2. Therefore we have to adapt the strategies such that at least the payoff (5,7) will be a possible solution and such that a sufficient number of solutions more or less close to (5,7) are also possible.

There seems to exist no specific evidence available from conducted experiments, which might support the hypothesis. Siegel and Fouraker (7, p. 61) have conducted experiments to test the effect of aspiration levels on the outcome of bilateral monopoly. The different possible maximum utility gains could induce different aspiration levels about possible results. However their experiments are not comparable with the present set up, because, among other things, they were conducted under incomplete information of both parties.

How would the solution look like in an exchange diagram? In Figure 16 we have shown the initial positions of persons I and II on the same indifference curve exactly as in Figure 4. The only difference with Figure 4 is that, moving from A to B on the contract curve person I increases his utility by 10, but person II by 20.

The point C shows the competitive solution. It is also the solution if the increase in utility for both persons is the same, as was the case in the previous section. But C can not be the solution if the maximum possible utility gains are different, according to the hypothesis. At C the utility gain of person I is 4 and the utility gain of person II is 8. In order to get a distribution of utility gains which is still Pareto optimal, but with an utility gain of 5 for person I and 7 for person II, person I has to move along the contract curve in the direction of B and person II in the direction of A such that exchange still remains possible.

The positions of persons I and II such that the utility gain of person I is 5 and of person II is 7, we have indicated by the points H_I respectively H_{II}. The price or exchange ratio at this solution is indicated by the slope of the vector $(H_{II} - II)$, which is equal to the slope of the vector $(H_I - I)$. Furthermore $(H_{II} - II) + (H_I - I) = 0$. Person II will exchange (b - a) of commodity 1 for (d - c) of commodity 2.

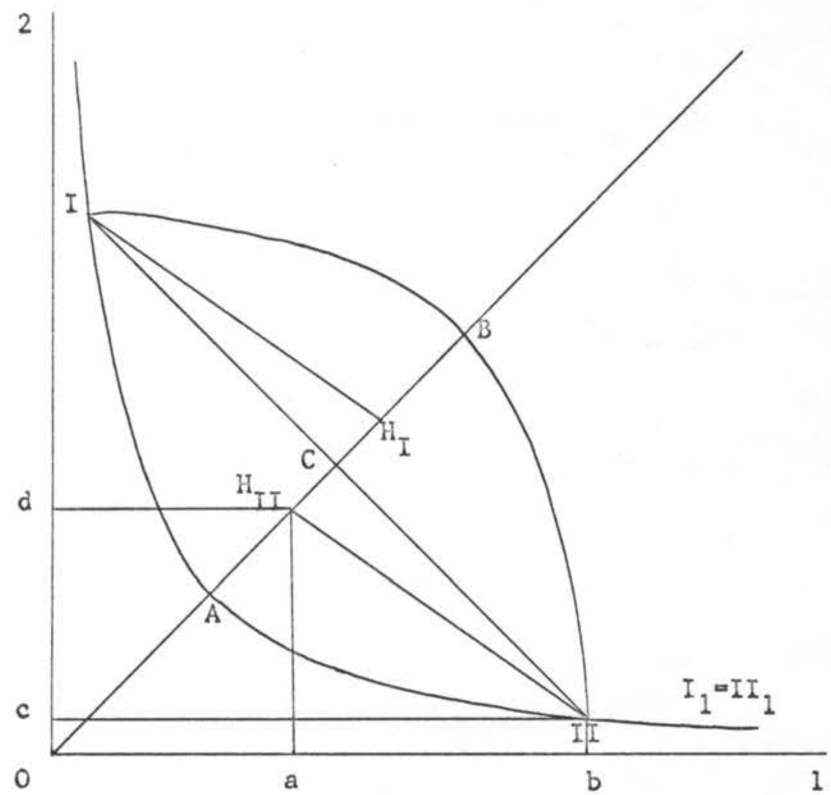

Figure 16. The solution in the exchange diagram

D. Unequal Initial Positions

Now suppose we increase the asymmetry of the situation for persons I and II still more by allowing for different utility levels at the initial position. In Figure 17 we have shown the positions of persons I and II at different indifference curves. The competitive solution may be constructed by the same method as used in section C of Chapter II.

The only difference is that the line with Pareto optimal positions of persons I and II now consists of two parts, one part AB for person I and one part CD for person II. We have shown the solution points H_I and H_{II} in the case that the utility gains are as in section C.

Now suppose the utilities belonging to the initial positions are 10 for person I and 30 for person II and the set S of possible utility gains is as in Figure 15. We then construct a set S' of possible utilities transforming the origin of the set S as is shown in Figure 18.

Our problem is now: Can we extend the hypothesis such that it is applicable to the set S' also? Suppose we could. First of all we see that the northeast boundary of the set S' is not Pareto optimal over its entire length. This would already make the point E of formula 6 a doubtfull endpoint of the range of possible solutions. The point E for instance would be the intersection of the line through the point (20,20) with the northeast boundary. In this case the point E is the point (20,20). But the point (20,30) then could be reached without person I loosing anything.

In fact person II might never consider any solution which would yield him a total utility of less than 30. Likewise person I would never

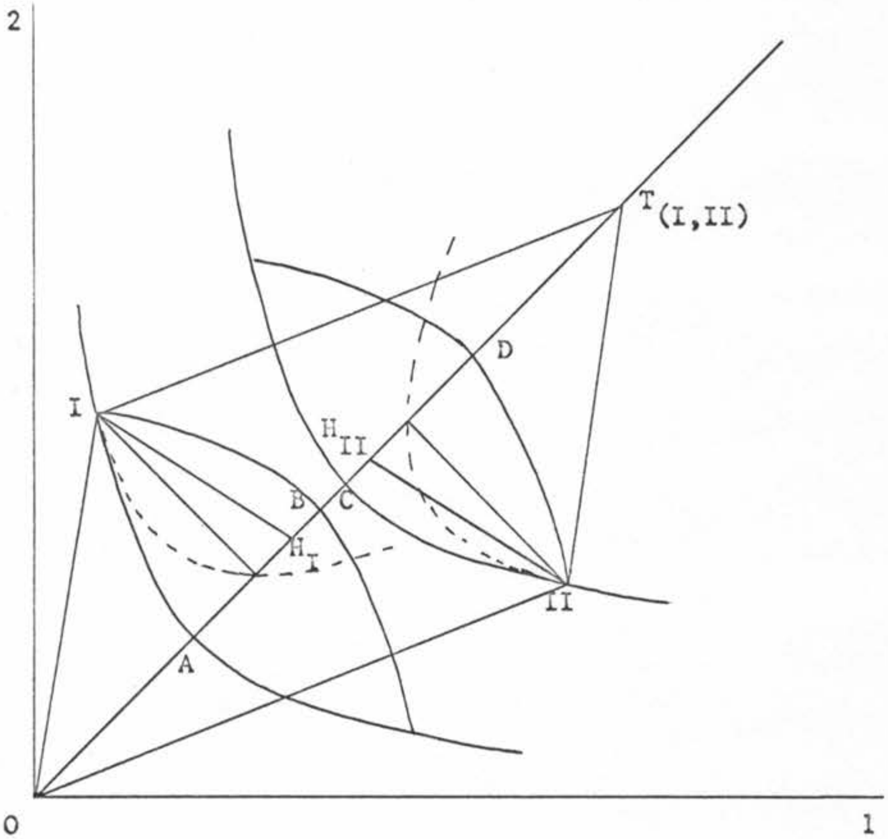

Figure 17. The exchange diagram for initial positions of persons I and II on different indifference curves

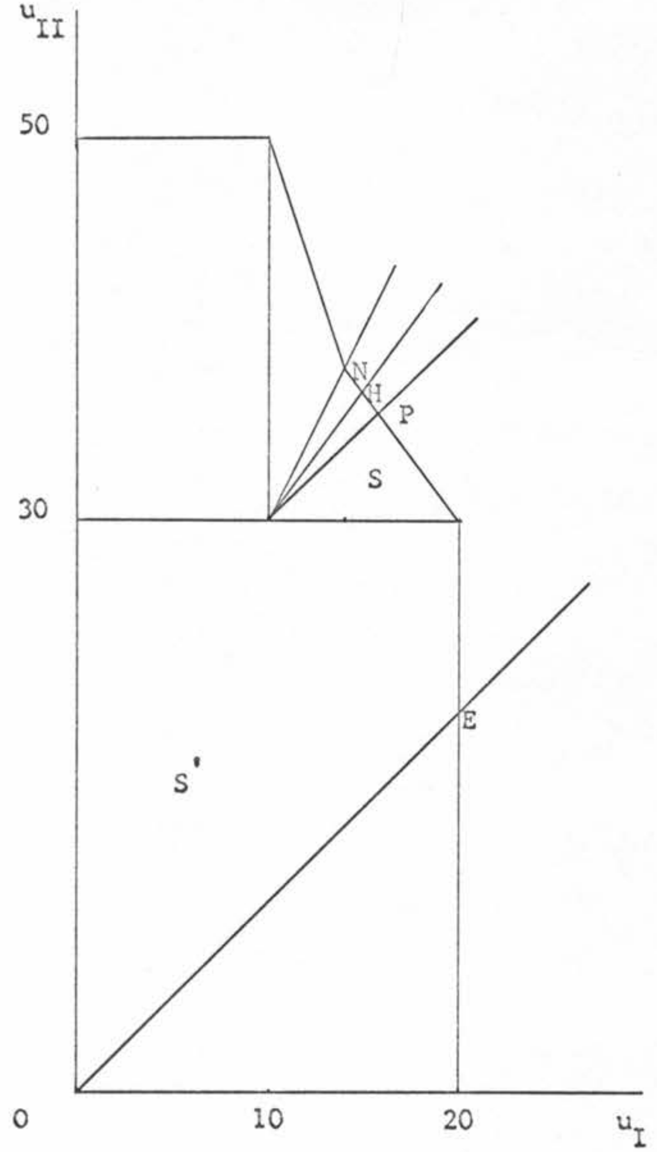

Figure 18. The set S' of possible utilities of persons I and II

consider a solution which would yield him a total utility of less than 10. Because the point (10,30) is the initial utility of both parties and neither one of them would likely agree upon reaching a position worse than his initial position. Furthermore nobody can force either person to a point worse than (10,30).

There is therefore a basic difference between the set S and the set S'. In the set S either party can force the other party to the point (0,0). In the set S' this is not possible. Either party can force the other party at most to the point (10,30). It is this reasoning which caused Nash (5) to consider the worst possible situation to which either party could be forced by the other party as the origin of the set S of possible utility gains. In this special case the origin of the set S would be therefore the point (10,30).

Therefore it is likely that the effect of interpersonal comparison of the utilities belonging to the initial states will be less than the effect of interpersonal comparison of differences of maximum possible utility gains. In the first case the parties can not force each other to a situation of equality, but in the latter case they can.

Still the initial positions may influence the solution in the set S. If the utilities of the initial positions have no effect, the solution will be the point $(\frac{34}{7},\frac{48}{7})$ in S. This solution was found as the point $\alpha(\frac{40}{7},\frac{40}{7}) + (1 - \alpha)(4,8)$, with $\alpha = \frac{1}{2}$. The effect of the interpersonal comparison of utility was expected to be a change in the solution from (4,8) if there was no interpersonal comparison of utility, to $(\frac{34}{7},\frac{48}{7})$, if there was interpersonal comparison of utility.

The hypothesis is now that the effect of interpersonal comparison of the utilities of the initial states will be, that the solution moves still further in the direction of $(\frac{40}{7},\frac{40}{7})$. Or in general, the solution found disregarding the utilities of the initial states, will be adapted in a direction more favorable to the person with lowest utility of the initial state. If the formula 6 of page 51 is:

$$\alpha E + (1 - \alpha)N \qquad 0 \leqslant \alpha \leqslant 1,$$

we may change it in:

$$\alpha E + (1 - \alpha)N \qquad \tfrac{1}{2} \leqslant \alpha \leqslant 1, \qquad 7$$

if the difference in utility of the initial position is favorable to person II. The more the difference in utility of the initial positions is in favor of person II, the closer α will approach to 1. That is, the solution will approach an equal division of possible utility gains.

The hypothesis may be tested again by playing the same type of games as suggested in the previous sections. But now at the beginning of the game each party is given an amount of money representing the utility belonging to his initial position. Both persons will be informed about the utility of each others initial position.

The effect on the solution in the exchange diagram of Figure 17 would be that H_I would move up still more along the line $OT_{(I,II)}$ and H_{II} more down along the same line.

With the adaption of the hypothesis in this section, the hypothesis can now take care of all possible two person two commodity exchange situations. Furthermore the hypothesis can be easily generalized to the m-commodity case. As the set S remains 2-dimensional, we can still derive a solution in it. But it will in general not anymore be possible to derive the solution in an exchange diagram, because the diagram will also become m-dimensional.

V. THE THREE PERSON CASE

A. Introduction

In this chapter we will extend our hypothesis to isolated exchange situations between three persons. This extension will introduce two new features which the two person case did not possess. We saw in the two person case that at any solution the exchange ratio between commodities was the same for both persons. This is due to the fact that at any change in the position of one person, the position of the other person must change by the same amounts.

In the three person situation this need not be the case. The only thing necessary is that the total sum of changes in position must add up to zero. This implies the possibility of different exchange ratios for the same pair of commodities as we will show more clearly later on.

Furthermore exchange in the two person case would only take place, if all the persons agreed upon the particular solution. In the three person case, exchange may take place, if only two of the three persons will agree about a particular solution, such that the third person is excluded from exchange.

It will turn out that these new aspects will have influence on the form of the set S of possible utility gains. Accordingly we will divide the discussion in four parts in order to be able to discuss each different feature separetely in relation to its effect on the set S. We will distinguish the following situations:

(1) No unique price but agreement between three persons necessary.

(2) Unique price and agreement between three persons necessary.

(3) Unique price and no agreement between three persons necessary.

(4) No unique price and no agreement between three persons necessary.

With unique price we mean that the exchange ratio between two commodities must be the same for all persons. Finally we will also consider the competitive solution and the restrictions on the set S required by it.

We will always start by assuming that the persons have all the same indifference curves. The indifference curves are orthogonal hyperbolas as in the previous chapter.

As there are now three persons the set S of possible utility gains will be three dimensional. It will have as boundaries the planes through the axes and a surface connecting these planes in the positive octant. For convenience we will call this surface also the northeast boundary of the set S.

We have to assure that this northeast boundary is Pareto optimal over its whole surface. This will be guaranteed again if the constructed utility functions of all persons are continuous and strictly increasing to the right. Depending upon the particular conditions under which exchange must take place, there are different possible contract curves or possibly even contract planes. However all these contract curves must have increasing values for each person moving to the right. As any movement of one person at any point of his contract curve must be offset by a movement of one or both of the other persons in opposite direction along their contract curves in order for exchange to be possible, an increase in utility along the contract curve for one person must imply a decrease in utility for at least one other person. This implies Pareto optimality. As the contract curves are continuous the decrease or increase in utility along it is continuous. Therefore the northeast boundary of the set S is continuous and Pareto optimal over its whole surface.

B. No Unique Price but Agreement between Three Persons Necessary

In Figure 19 we have shown the positions of persons I, II and III on the same indifference curve. The total amount of commodities 1 and 2 has always to add up to $T_{(I,II,III)}$. We suppose further again for simplicity that the indifference curves of persons I, II and III are exactly the same and are orthogonal hyperbolas.

The curve III_3 of maximum possible utility gains of person III for persons I and II remaining on their initial indifference curves, such that exchange will be possible, may be found as follows: Let II move along the indifference curve to person I. Then III has to move down along the curve m to the point P in order for exchange to be possible. Now let persons I and II move together down along the indifference curve. Then III has to move along the curve III_3 in order for exchange to be possible. If persons I and II have arrived at Q, III will have arrived at R, the distance PR being two times the distance from I to Q. Furthermore, if persons I and II have arrived at the initial position of person III, III will have arrived at S. III will never be able to get more to the right, because at any point on III_3, any movement of

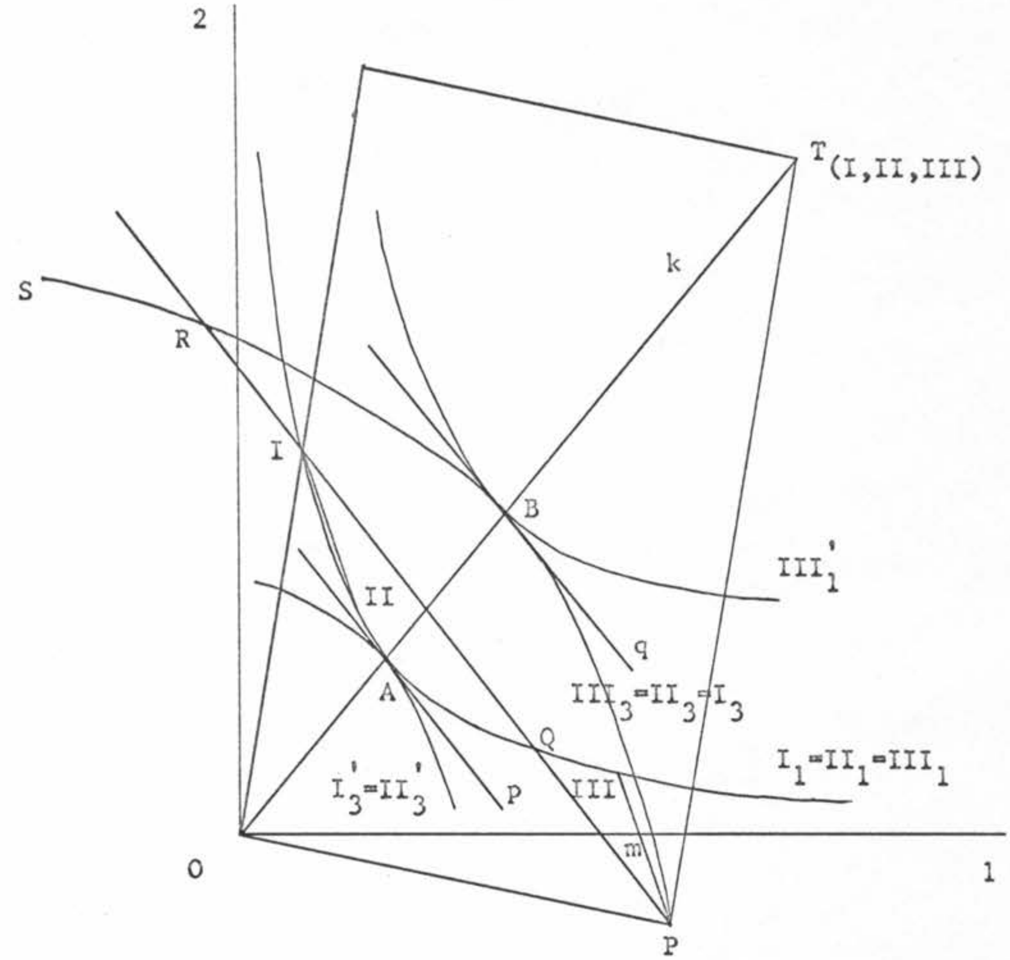

Figure 19. Construction of the curve of maximum possible utility gains

persons I and II together will keep III on III_3 and any movement of person I or II alone will bring III to the left of III_3.

Now it can be shown that III_3 is identical with I_3 and II_3. Because if persons I and III interchange their position, the total amount of commodities will still add up to $T_{(I,II,III)}$. Now we can use the same procedure with the roles of persons I and III interchanged. The same is true for the curve II_3.

As far as exchange possibilities are concerned, only the region enclosed between the initial indifference curve and the curve III_3 is of interest, because for points outside this region at least one party will be worse off than at his initial position. What can we say about the Pareto optimal positions within this region?

The highest possible utility for any person such that the other persons remain on their initial indifference curves, is now indicated by B. B is the point of tangency of an indifference curve with the curve $III_3 = II_3 = I_3$. Furthermore B lies on the straight line k from O to $T_{(I,II,III)}$. If person III is at B, persons I and II must be at A, which lies also on the straight line k. That A must lie on the straight line k, if B lies on it, follows from the addition properties of vectors. In particular it can be shown to follow from the fact that the diagonal from I to P is cut by the line k in parts with ratio 1 : 2.

From the properties of orthogonal hyperbolas we know, that any straight line from the origin intersects the hyperbolas at points with parallel tangent lines. We call the tangent line at A, p and at B, q. If persons I and II move along p, person III must move along q in order for exchange to be possible. As p has only one point in common with the indifference curve $I_1 = II_1$, q can only have one point in common with III_3. q can not intersect III_3, because then p must intersect $I_1 = II_1$. Therefore at B, q has one point in common with III_3 and with III_1', so B must be the point of tangency.

In the same way it can be shown that the straight line AB is the set of Pareto optimal positions for persons I, II and III. In this case also therefore the contract curves of persons I, II and III are the same. This does not imply however, that the positions of persons I, II and III are always the same on this contract curve, as we have seen already. In fact there is only one point on the contract curve at which the positions of persons I, II and III are the same and such that exchange is possible.

We may show this in Figure 20, which has the essential aspects of Figure 19. Suppose person III moves from B to C along the straight line AB. Then persons I and II have to move together half the distance in opposite direction in order for exchange to be possible, say to D. The points C and D are a Pareto optimal combination for persons I, II and III. But persons I and II need not move together to D. Person I may stay at A and person II may move an equal distance as III but in opposite direction, say to F. This is again a combination of Pareto optimal positions.

In fact any movement of persons I and II along AB, such that the sum of their displacements is equal in absolute value to the displacement of person III, will satisfy the condition of Pareto optimality. At the same time any movement of persons I and II not along AB, such that the sum of

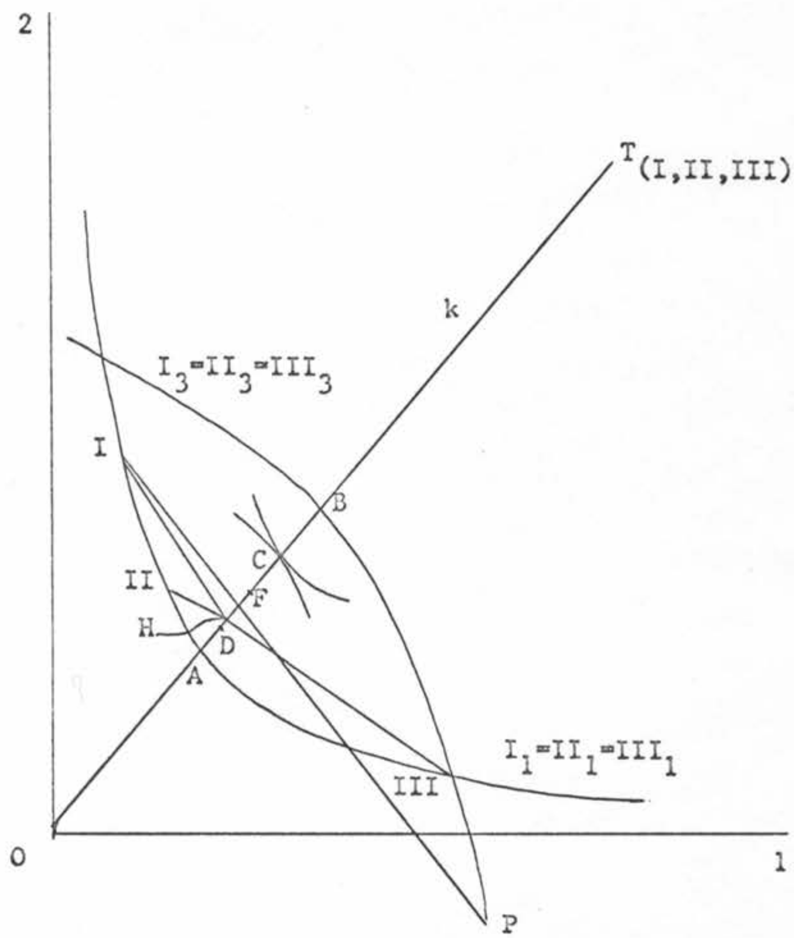

Figure 20. The unique point H on the contract curve

their displacements adds up to a point on AB, is seen not to be Pareto optimal. Because it can always be replaced by a movement of both persons along AB, such that the sum of the displacements adds up to the same point as before but with the utility of at least one person increased and of nobody decreased.

Now suppose person III moves downward along AB and persons I and II move upward together. Then there must be one point along AB at which the positions of persons I, II and III are the same. Furthermore there can be only one point because otherwise there would exist two vectors such that, if they were both multiplied by 3, they both would result in the same vector $T_{(I,II,III)}$. This is impossible. We have shown this unique position of persons I, II and III as the point H in Figure 20. H is the intersection of the diagonal from I to P with the line k.

We see that if H is the solution point of the three person exchange situation, persons I, II and III will all exchange commodities 1 and 2 for different exchange ratios. The amounts exchanged and the exchange ratios are determined by the vectors (H - III), (H - II) and (H - I). Furthermore (H - I) + (H - II) + (H - III) = 0.

How will the set S of possible utility gains look like for this particular exchange situation? Let us suppose first that the maximum possible utility gain of each person at the point B in Figure 20 is the same, say 10. The particular form of the northeast boundary of the set S depends upon the utility curves for each person belonging to the contract curve AB. If the utility curves for all persons are linear for the contract curve, then the northeast boundary of the set S is a plane as shown in Figure 21.

In general however the northeast boundary may have all possible forms such that it still remains Pareto optimal. If the utility curves belonging to the contract curve AB are convex for all persons for instance, the northeast boundary will also be a convex plane with respect to the origin.

We will now extend the hypothesis to deal also with the case of three person exchange, where agreement between three persons is necessary, but not necessarily at a unique price. Considering the set S of Figure 21, we see that any person can force a solution (0,0,0), which is analog to the situation in the two person case. Likewise we may repeat similar arguments for the case of unequal maximum possible utility gains.

One could argue that this three person case is basically different because of the possibility of cooperation. However cooperation has only sense if one party can be excluded in some way or another. This possibility is excluded in this section. That is, as any person has to agree about the solution, any person can always force a solution (0,0,0) if he thinks that the proposed solution is unfair to him.

In general therefore, if we call the endpoints of the Pareto optimal northeast boundary of the set S, A, B and C and if we call the intersection of the line through (A,A,A) with the Pareto optimal surface, E and the intersection of the line through (A,B,C) with the Pareto optimal surface, N, the solution according to the hypothesis will be the intersection of the Pareto optimal surface with the line through the point:

$$\alpha E + (1 - \alpha)N \qquad 0 \leqslant \alpha \leqslant 1 \qquad 8$$

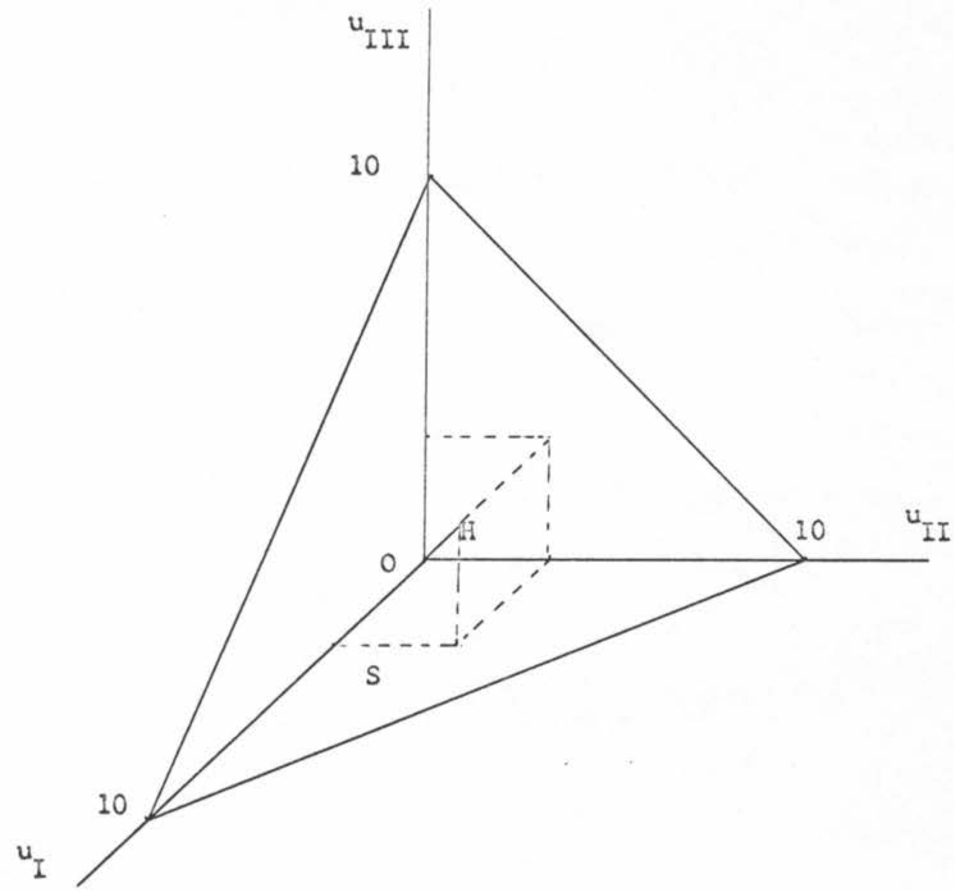

Figure 21. The set S of possible utility gains for linear utility curves belonging to the contract curve

For purposes of exposition we may again take $\alpha = \frac{1}{2}$ to find unique solutions. In the case of Figure 21 the points E and N will be the same and the solution point H can be found as $(\frac{10}{3}, \frac{10}{3}, \frac{10}{3})$.

We generalize the hypothesis to the m-commodity case for any positions of persons I, II and III on any indifference curve and for any strictly increasing utility function of each person. The exchange diagram would become m-dimensional also, but the set S would still be 3-dimensional.

To test the hypothesis we may construct a payoff "matrix" for the game. The payoff "matrix" for the exchange situation represented by Figure 19, will now be a cubic with every horizontal slide looking like Figure 13. That is, any triple with at least one non-negative entry will be the only one of this type in the same horizontal row and column and in the same vertical column. The representable strategies should be chosen such that possible alternative solutions are included.

No preplay communication is allowed and all players are informed completely about the strategy sets and payoffs of everyone. The payoffs will be in money and randomization among prospective players would exclude other effects.

The game will be played as follows: Person I announces a strategy to persons II and III. Then person II announces a strategy to persons III and I and then person III to I and II. The game is ended if the strategies announced by all players have a non-negative payoff for at least one person and the next round of announcements is exactly equal to the first round. This last procedure is necessary to make the positions of the persons symmetric with respect to the bidding.

If the next round of announcements is not exactly equal to the first, the bidding continuous. The game is ended if the announced strategies are repeated once and have a non-negative payoff for at least one person, or the game is ended after a definite amount of time. If agreement is reached about exchange, it takes place and the parties receive the payoffs.

Differences in utilities of the initial positions may be taken care of by giving the players the money payoff belonging to the utilities of the initial positions at the beginning of the game. Every player again is informed about these initial payoffs.

C. Unique Price and Agreement between Three Persons Necessary

Let us now investigate what happens if it is necessary that the exchange ratio is unique. In Figure 22 we have shown the positions of persons I, II and III on the same indifference curve. We assume again that all persons have the same indifference curves which are orthogonal hyperbolas. First of all let us construct the curves of maximum possible utility gains.

We can not apply the same procedure as in the previous section, because that involves already different prices. Let us first construct the curve of maximum possible utility gains for person I. If person III moves to A on his initial indifference curve, person I can move to B, if person II stays at his initial position. Furthermore person I can not move more to the right, because then either II or III would reach a position with an utility loss. So B is on I_3. In the same way we can

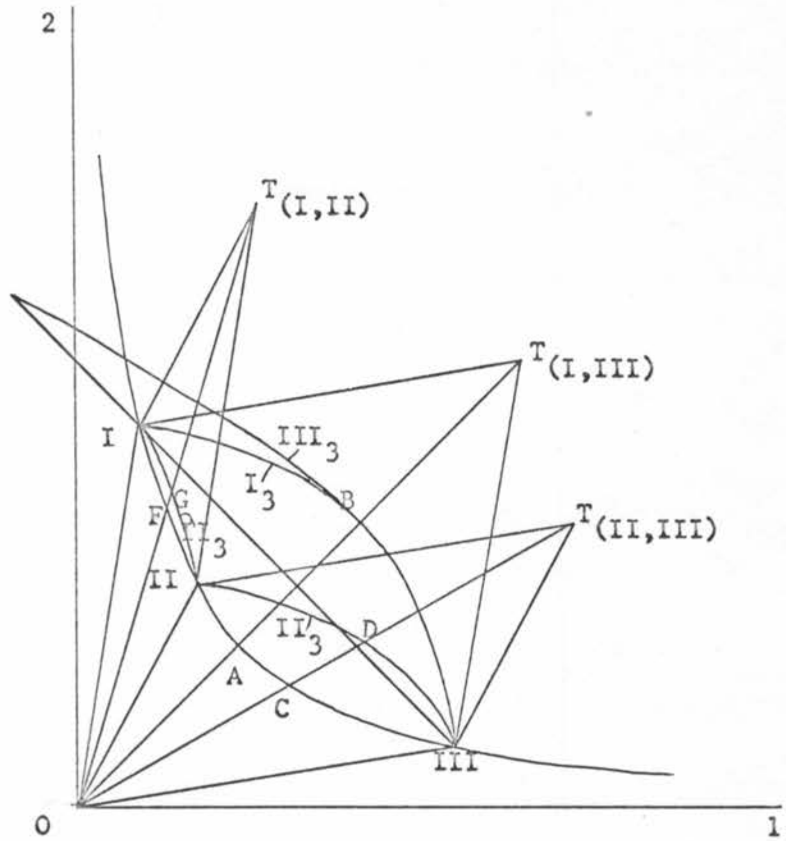

Figure 22. The curves of maximum possible utility gains for persons I, II and III

construct the whole curve I_3 for person III moving along his initial indifference curve and II staying at his initial position.

The curve III_3 may be constructed in the same way. But now not only person I may move along the indifference curve but II also, such that the change in positions of persons I, II and III occurs along parallel lines. Therefore at the top the curve III_3 is a bit different from I_3.

The curve II_3 consists of two parts, one for person III moving along his initial indifference curve and person I staying at his initial position and one for person I moving along his initial indifference curve and III staying at his initial position.

By the properties of the orthogonal hyperbolas, the straight lines AB, CD and FG are contract curves if only the persons I and III, or II and III, or I and II exchange with each other. From these contract curves we can find the endpoints of the northeast boundary of the set S. For persons I and III this is at the point B with utility gain of say 10. For person II it is at D with utility gain of say 6. In Figure 23 we have shown these endpoints of the set S.

In general it will be difficult to find the exact form of the Pareto optimal northeast boundary of the set S. At present we do not know of a general method to derive curves or planes of Pareto optimal positions for situations like the one shown in Figure 22 and therefore we can not derive particular boundaries of the set S. But we know that the northeast boundary is Pareto optimal. For simplicity therefore we assume that the northeast boundary has the form shown in Figure 23. In any particular situation the northeast boundary of the set S may be approximated as

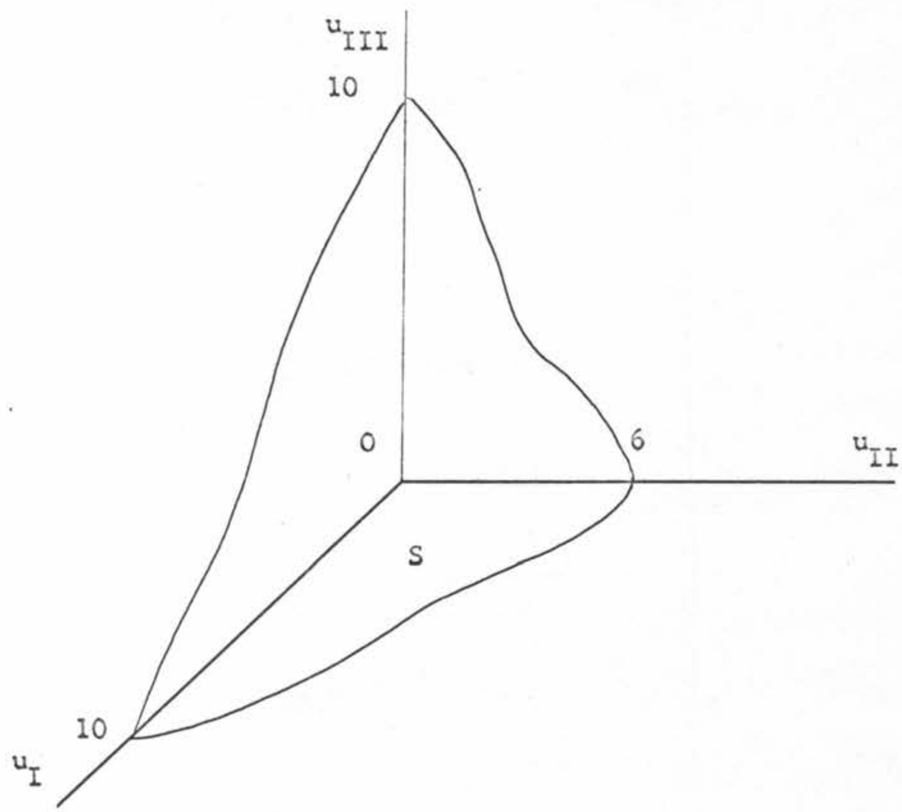

Figure 23. The set S of possible utility gains

closely as necessary directly from the utilities belonging to any possible solution in Figure 22.

The set S in Figure 23 represents the exchange situation of Figure 22 for the particular positions of persons I, II and III. But the set S will change if the positions of the persons change. Of particular interest is the change in the set S, if the position of person II along the initial indifference curve changes. Without going into detail it may be seen that the general shape of the set S, if person II has the same position as person I, is as in Figure 24a. See for this figure for example also Figure 27. On the other hand if the position of person II is at A in Figure 22, the general shape of the set S will be as in Figure 24b.

We now extend the hypothesis to include also the general case of isolated exchange between three persons with agreement necessary between all persons at a unique price. We may use the hypothesis and the formula 7 of the previous section directly for this section. The solution will still be different in general, because it can be shown that the set S of this section will always be contained in the set S of the previous section.

The points E and N necessary for the solution, will be found most easily by looking in Figure 21 for a utility combination on the Pareto optimal northeast boundary, which satisfies the parametric expressions (6,6,6) and (10,6,10) for a particular and for E and N respectively.

In Figure 22 the solution points for each player will be such, that the movements of all persons from their initial positions to their positions at the solution will be along parallel lines through the initial positions of the persons.

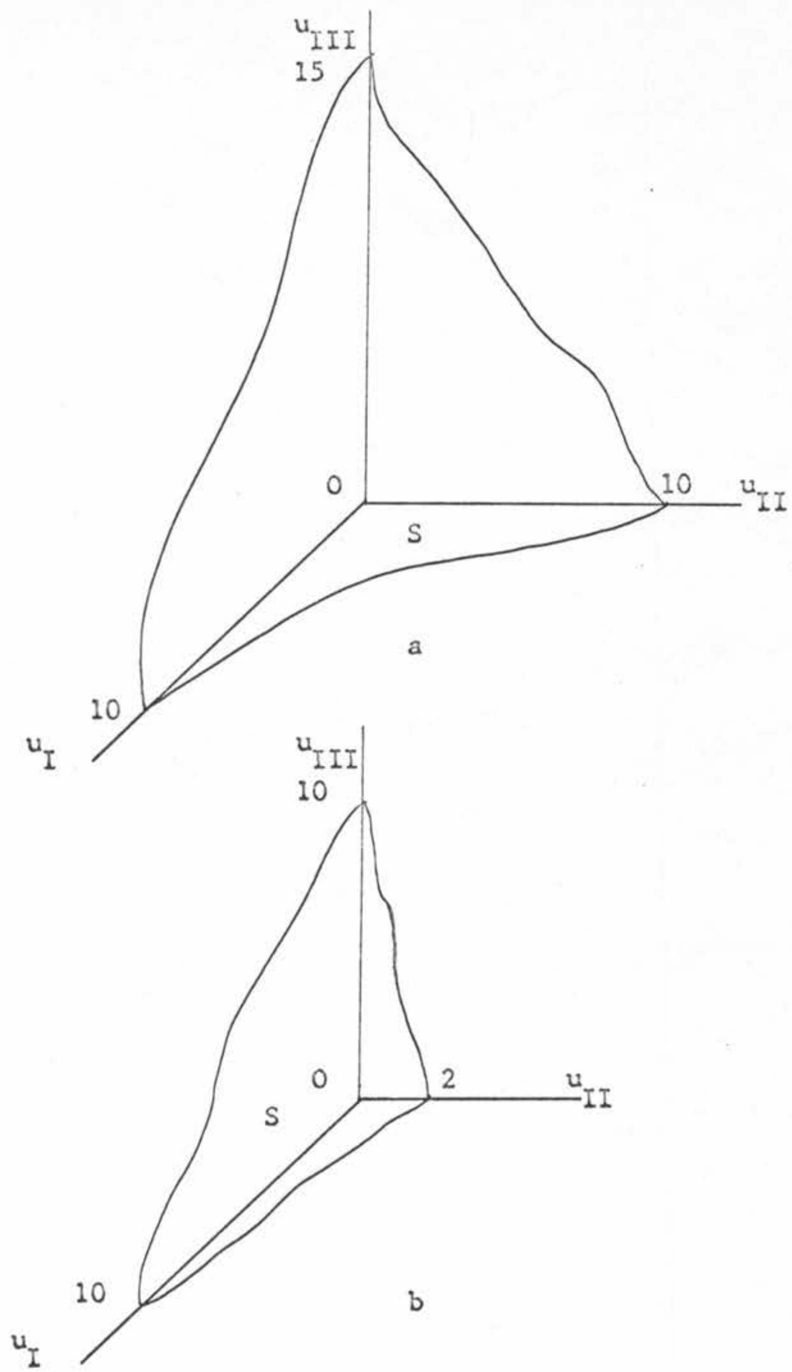

Figure 24. a. The set S for equal positions of persons I and II

b. The set S for II in the middle between persons I and III

D. Unique Price and No Agreement between Three Persons Necessary

Let us consider the same situation as in the previous section, but now we allow exchange to take place at a unique price if agreement between two parties is reached. Suppose the initial positions and conditions are as in Figure 22.

We may now distinguish two possible sets S depending upon whether agreement between three parties is reached or between two. The set S of possible utility gains if agreement is reached between three parties, will be exactly the set S of Figure 23. On the other hand, the set S of possible utility gains if agreement only between two persons is reached, may be found by considering again Figure 22.

If agreement is reached between two persons, then the third person is excluded from the exchange and his utility gain will be zero. This implies that the set S can have no points with all coordinates positive. In other words the set S must consist only of parts of the planes through the axes of two of the three persons. The upper boundaries in these planes may be easily found by considering the three possible two person exchange situations of Figure 22.

Using the same utilities as in the previous section, we see that the maximum possible utility gain for either person in the exchange situation between persons I and II, is 1 at the point G, between persons I and III it is 10 at the point B and between persons II and III it is 6 at the point D. For simplicity we assume the utility functions of the persons to be linear above the contract curve. The resulting set S will then be as in Figure 25.

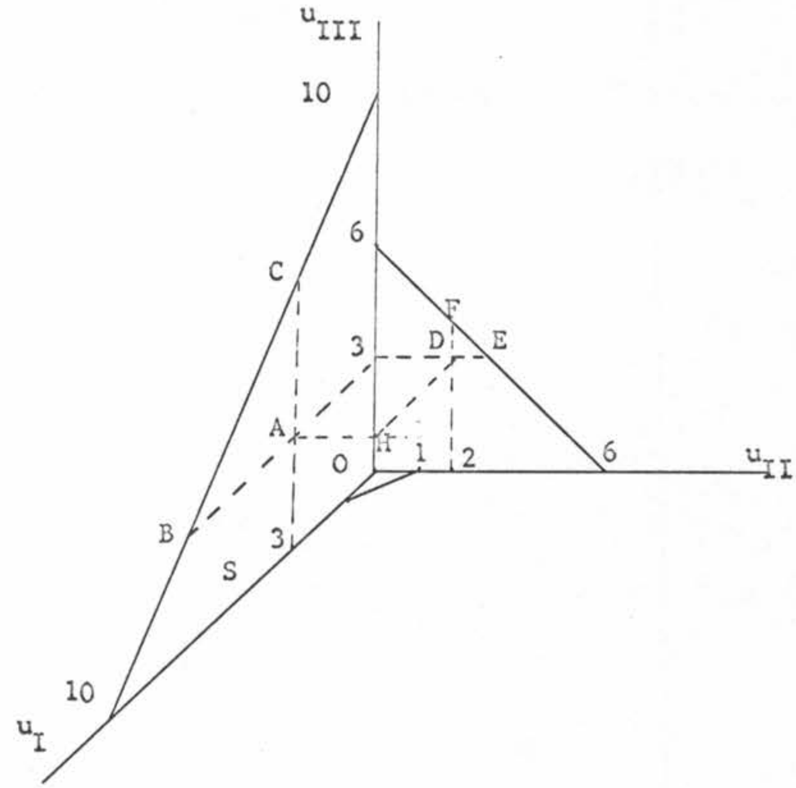

Figure 25. The set S if agreement is reached between two persons at a unique price

Suppose the solution point in the set S of Figure 23 was H = (3,2,3) in the previous section. Suppose this point was reached in the repeated play of a game. But suppose now the rules of the game are changed such that at the end of the game the parties are allowed to continue the bidding and the game will be ended also if only two parties announce repeatedly a strategy such that between them exchange can take place with the exclusion of the third party. What will happen?

The point H in S is Pareto optimal with respect to the utility gains of three parties. However it is not Pareto optimal with respect to the utility gains of all possible couples of two parties as may be seen in Figure 25. For this particular solution it is not Pareto optimal with respect to the two person exchange situations between persons I and III and between persons II and III. Furthermore person III will be able to increase his maximum possible utility gain more by exchanging with person I than by exchanging with II, as may be seen by comparing the triangle ABC and DEF.

But this does not naturally imply that person III will exchange with person I. Because person II may fear to be excluded with the result of gaining nothing and may therefore offer a more favorable bid for III in the game.

However person II will not make any possible offer. Consider again the situation of Figure 22, which we have shown now in Figure 26. If persons I and III exchange with each other with person II having no effect on the solution, the solution for persons I and III will be the point $H_{(I,III)}$. If person II wants to prevent this solution he has to make a

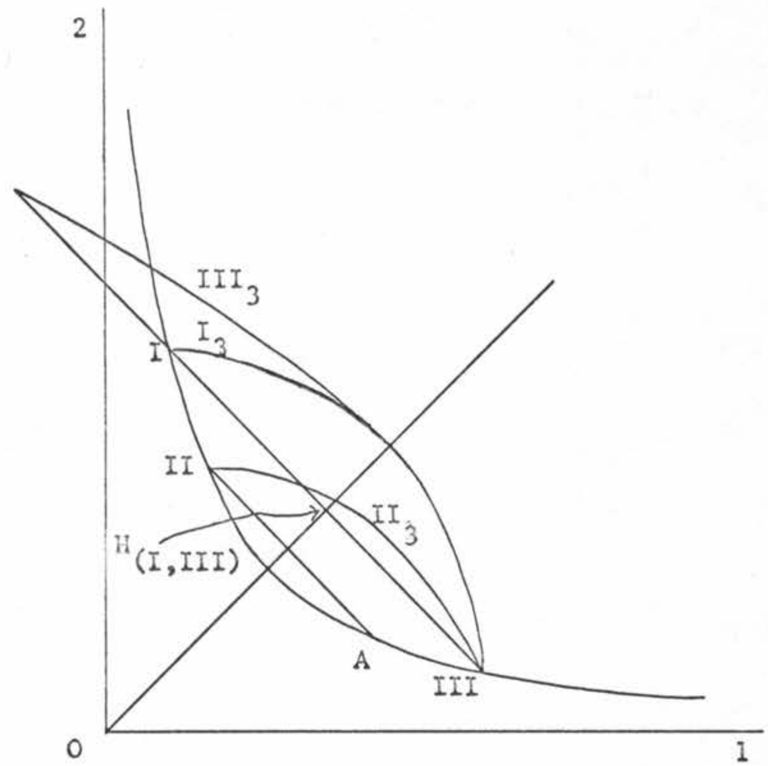

Figure 26. The possibility of person II exchanging with person III

bid which will change his position at least to the left of the point A on his indifference curve. The distance from II to A being the same as the distance from I to $H_{(I,III)}$. Because only then III will move his position to the right of $H_{(I,III)}$ and gain more utility. But then II will reach a position worse than his initial position. Therefore in this case person I has apparently nothing to fear and the solution will be $H_{(I,III)}$ with II remaining at his initial position.

But now suppose that the positions of persons I and II are the same as is shown in Figure 27. Then the solution would be $H_{(I,II)}$ and H_{III} say, if all three persons had to agree about a unique price. But if only two persons could exchange, then the two persons could certainly increase both their utility gain by agreeing upon solutions H_I and H'_{III} for persons I and III for instance. With this possibility either person I or II would not gain anything and III apparently would be in the most favorable position.

If we would construct a payoff matrix for the persons I and II alone, it would turn out that it would have similar characteristics as the payoff matrix of the game called "the prisoner's dilemma" (3, p. 94). Suppose person III plays no active role for the moment. Then we can see that there is a great temptation for both persons I and II to deviate from the strategy which results in the solutions $H_{(I,II)}$ and H_{III}. Because at first sight a strategy offering the solutions H_I and H'_{III} or H_{II} and H'_{III}, increases the payoff for person I or II. However the other person may retaliate with the ultimate effect that the solution reached will be less favorable than the solution at the position $H_{(I,II)}$.

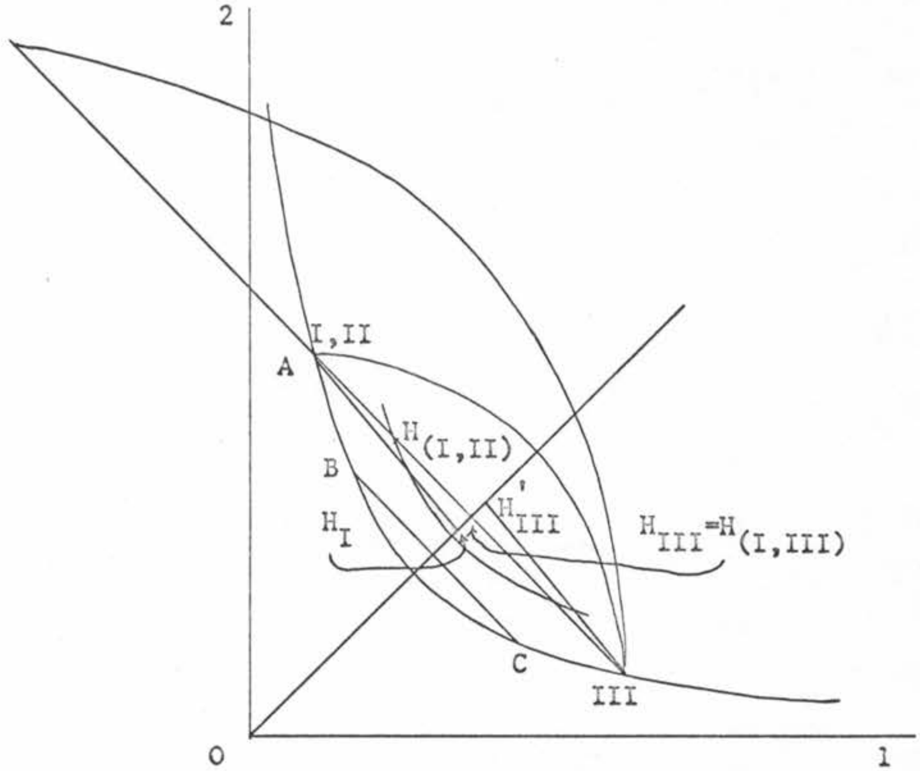

Figure 27. Possible solutions if the positions of persons I and II are the same

Therefore after some initial ventures persons I and II may decide at last that $H_{(I,II)}$ is the best solution and stick to the strategies necessary for this solution. As Luce and Raiffa argue, this solution is likely if the play of the game is repeated indefinitely. However there is one difference and that is the role which person III may play. The effect of the bids of III will certainly be that the solution point $H_{(I,II)}$ will be still more unstable. However any alternative person I can offer, II can offer as well because of their symmetric positions. We suspect therefore that the points $H_{(I,II)}$ and H_{III} will still be the solution.

It may be remarked that III could still play a strategy for the solutions H_I and H'_{III} by agreeing only to deal with person I. Person I would possibly agree with that. However II can always offer exactly the same bid as person I. In that case the rules of the game can not decide between persons I and II and the game is not ended.

So we have as the hypothesis for this case, that if the positions of persons I and II are at point A, the solution will be the same as in the previous section, namely the points $H_{(I,II)}$ and H_{III}, determined by applying formula 8 of page 84 to the set S of Figure 23. On the other hand if the position of II is such that he can not reasonably threaten the position of either player I or III, that is, if his position is between B and C in Figure 27, then the solution will be the point $H_{(I,III)}$ with II remaining at his initial position. The point $H_{(I,III)}$ is determined by applying formula 6 of page 51 to the set S of possible utility gains of persons I and III.

If II moves from A to B along the indifference curve of Figure 27, his effect on the solution decreases such that the position of person I in the solution moves from $H_{(I,II)}$ to $H_{(I,III)}$, and the position of person II from $H_{(I,II)}$ to B.

If we call the solution point in the set S of Figure 23 H_a if the initial positions of persons I and II are at A and H_b if the initial position of person II is at B, then we can use as the hypothesis, that the solution will be the intersection of the northeast boundary of the set S of Figure 23 with the line through the point:

$$\alpha H_a + (1 - \alpha)H_b \qquad 0 \leqslant \alpha \leqslant 1 \qquad 9$$

If the initial positions of persons I and II are at A, $\alpha = 1$. If the initial position of person II is between B and C, $\alpha = 0$.

We may apply therefore to this solution the same considerations of interpersonal comparison of utility, if the maximum possible utility gains of say III and I are different. However, we should take into consideration that a change in the solution may change also the range over which the other party can threaten the solution.

For instance, if in Figure 27 the maximum increase in utility of III is 20, the solution will not anymore be the point $H_{(I,III)}$. Instead the solution point for person I will be more upward and to the right and for III more downward and to the left. This implies that the point B, at which II is not anymore threatening person I, moves downward along the indifference curve. If these factors are taken into consideration, the hypothesis may be applied for any initial positions, any increasing utility curves and for m-commodities.

E. No Unique Price and No Agreement between Three Persons Necessary

It might seem that this case is already dealt with, because if there is no unique price, then there can only be agreement between two persons and we are back in section B. But there is one difference. In section B we investigated the situation where any player had the power to exclude unfavorable solutions. This is not the case now.

We may also see the difference between this section and the previous section as follows. Instead of comparing the set S of Figure 23 with the set S of Figure 25, we now compare the set S of Figure 21, with the set S of Figure 25. Or we may compare the solutions in Figure 22 and Figure 27, with the solutions in Figures 20 and 27.

Suppose the positions of persons I, II and III are as in Figure 28. Under the usual initial conditions, the solution of the previous section would allow then say that person I moved to A, II to C and III to E. The possibility of different prices now allows also for a movement of person I to B and of II to D such that there is still exchange possible. As the possibilities of section D are included in section E, this may imply that the sum of the utility gains of all persons is increased relative to the situation in section D. Because as the set S of Figure 21 includes the set S of Figure 23 the sum of the utility gains of the Pareto optimal solution may increase.

The hypothesis is now therefore the same as in the previous section. But instead of using the set S of Figure 23 we use now the set S of Figure 21 for our considerations and in particular for the application of formula 9.

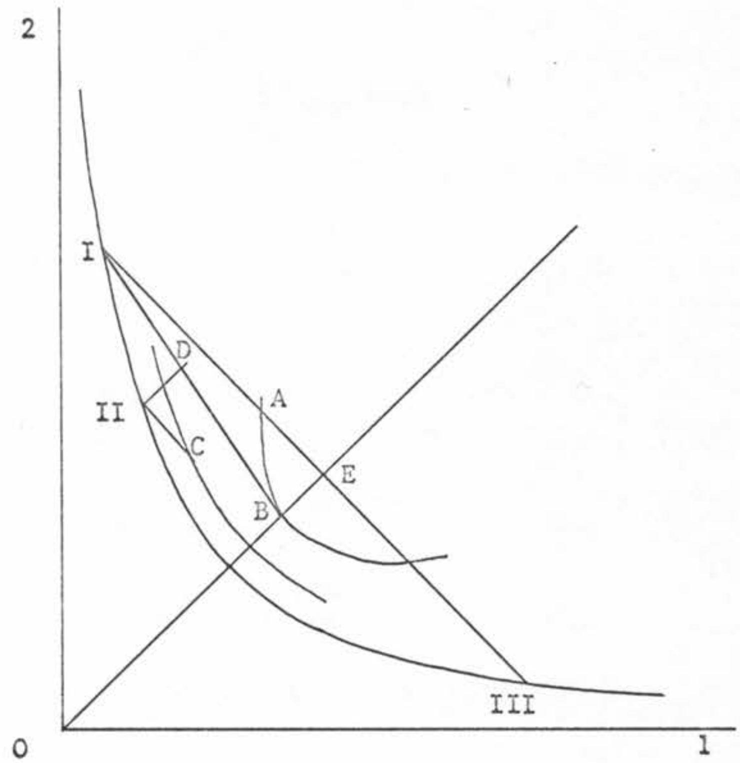

Figure 28. The effect on the solution if no unique price is necessary

F. The Competitive Solution

Finally let us consider the competitive solution of the three person two commodity case. In Figure 29 we have shown the three positions of persons I, II and III on the same indifference curve. The curves I_2, II_2 and III_2 are the trading curves of persons I, II and III for all relevant prices.

The competitive solution may be found by constructing parallel lines through the positions of persons I, II and III such that the sum of the changes in positions adds up to zero. We have shown the positions of persons I, II and III at the competitive solution by C_I, C_{II} and C_{III}.

For any positions of persons I, II and III on any indifference curves in the plane of commodities 1 and 2, the competitive solution may be found by using the same procedure. Special cases are again the situation where persons I and II are both at the point A, in which case the solution for both will be the same point under the usual special conditions and the case where person II is at B, in which case he will not take part in the exchange.

The set S of possible utility gains is the set S of Figure 23 under the same restrictions for the utility functions as required in section B. The competitive solution is Pareto optimal, but we can not find it directly in the set S. We may find it in the set S of Figure 23 by taking the utility gains of each person belonging to the competitive solution in Figure 29.

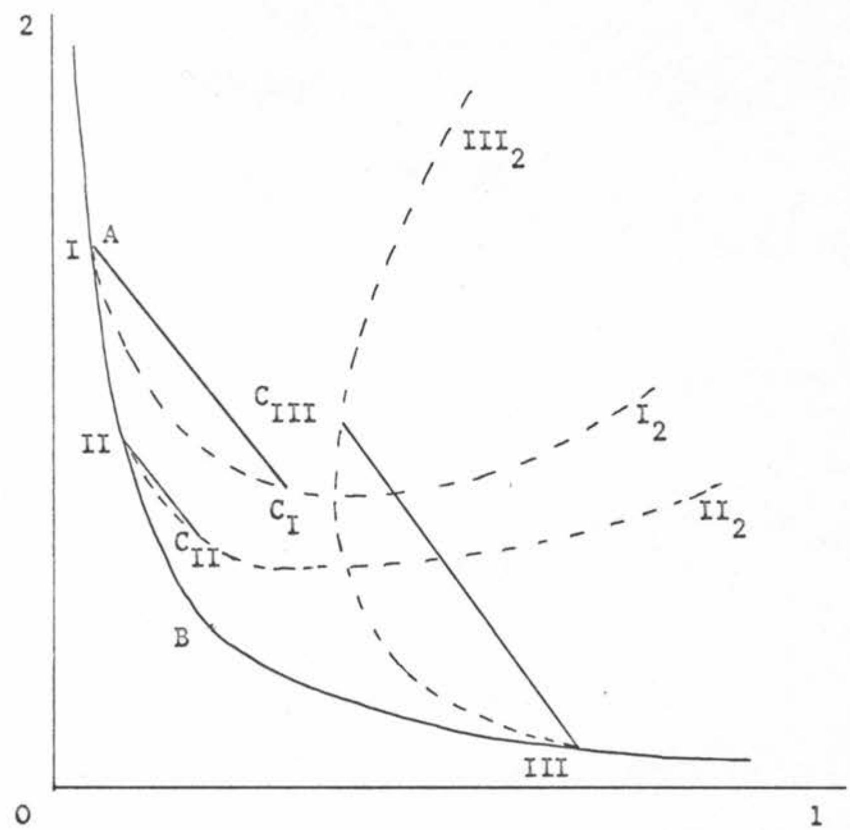

Figure 29. Construction of the competitive solution

VI. SUMMARY

As a point of departure for the investigation of the effects of interpersonal comparison of utility on the solution in isolated exchange situations, in Chapter II the competitive solution and the Nash solution are discussed and a comparison is made between these two proposed solutions. In order for a comparison to be possible, it is necessary to assume that utilities can be measured through a linear utility assignment scheme, satisfying the Von Neumann and Morgenstern axioms of utility theory.

It is found then, that the competitive solution does not satisfy several assumptions necessary for the Nash solution. In particular it does not satisfy the assumptions of independence of irrelevant alternatives and symmetry. Furthermore it is seen that the set S of possible utility gains, belonging to any particular isolated exchange situation, does not satisfy in general the condition of convexity required by the Nash solution. The competitive solution does satisfy however the conditions of invariance with respect to linear utility transformations and Pareto optimality.

In Chapter III first of all the conditions are stated, under which isolated exchange will be investigated. The isolated exchange situation will be considered as a special type of repeated play of a game, with complete information of the players. Particular effects of bargaining skills on the solution are excluded. It is assumed that the utilities of persons are representable by a linear utility assignment scheme satisfying the Von Neumann and Morgenstern axioms of utility theory and such that the utilities of all persons are measured with respect to the same origin and are multiplied by the same constant.

Based upon these conditions, the Nash solution is considered to be more likely than the competitive solution in the case that there is no interpersonal comparison of utility and the set S of possible utility gains is convex. By considering the criticisms of the Nash solution, then a special hypothesis for a solution is proposed for the case that interpersonal comparison of utility is possible and the set S of possible utility gains has a northeast boundary which is Pareto optimal.

The hypothesis is that, if the endpoints of the Pareto optimal curve of any set S, belonging to an isolated exchange situation in which the utilities of the initial amount of commodities are the same for both persons, are called A and B, and if the intersection of the Pareto optimal curve with the line through the point (A,A) is called E and with the line through the point (A,B) is called N, then the solution will be the intersection of the Pareto optimal curve with the line through the point:

$$\alpha E + (1 - \alpha)N \qquad 0 \leqslant \alpha \leqslant 1 \qquad 1$$

The exact value of α is supposed to be determined by experiment rather than by reasoning. For convenience of exposition however it is assumed that $\alpha = \frac{1}{2}$. The hypothesis is seen to satisfy only the conditions of Pareto optimality and symmetry.

In Chapter IV the two person isolated exchange situation is dealt with in detail as a game. From the exchange diagram a representable payoff matrix is derived with a finite number of strategies and with the utility gain of person I as the first entry and the utility gain of person II as the second entry in the payoff square belonging to a particular pair of strategies of persons I and II. From the payoff matrix or directly from the exchange diagram the set S of possible utility gains is derived.

The game is played as follows: No preplay communication is allowed. Player I starts the game by announcing a strategy to player II. Next player II announces a strategy to player I and so on. The game is ended if the strategies announced by both players have a corresponding payoff square with at least one positive entry. Or the game is ended after a certain amount of time has passed.

The hypothesis is adapted for the case of unequal utilities of the initial positions. The positions of the persons are indicated by a vector in the exchange diagram with coordinates the amounts of commodities of the persons. The solution, determined by formula 1, will be influenced favorably in the direction of the person with lowest utility of the initial position. After this generalization the hypothesis is supposed to be applicable to any two person m-commodity isolated exchange situation in which the persons have utility functions, which are strictly increasing in the same direction.

In Chapter V the three person isolated exchange situation is investigated. It is found that the addition of one person adds two new aspects to the problem, the possibility of more than one exchange ratio between two commodities and the possibility of exchange between only two of the three persons. Consequently the chapter is divided in four main sections depending upon whether agreement between all persons is required and/or a unique price is necessary.

Depending upon the different possibilities, the 3-dimensional set S of possible utility gains will also have different possible forms. If agreement between three persons is necessary but not at a unique price, it is argued that the case is analog to the two person case dealt with before. Consequently the hypothesis is extended to this case also. The only difference is that the points (A,A) and (A,B) used in applying formula 1, are now replaced by the points (A,A,A) and (A,B,C) in which A, B and C are the points of the Pareto optimal surface on the x-, y-, and z-axis.

If agreement between three persons is necessary at a unique price, the hypothesis is the same as before, but as the northeast boundary of the set S will in general be different, the solution will be different also. More specifically the sum of the utility gains of all persons at the solution, if a unique price is necessary, will never exceed the sum of the utility gains at the solution, if no unique price is necessary.

If a unique price is necessary but not agreement between all persons, the solution will depend upon the relative positions of the persons. The relative positions of the persons determine the influence each person can have on the solution. If the indifference curves of all persons are the same and if the initial positions of all persons are on the same indifference curve, such that the positions of two of the three persons are the same, then the solution will be determined by the same hypothesis as in the previous three person cases. If the position of one person moves to a point on the initial indifference curve in the middle between the two other persons, the effect of the person on the solution will decrease until after a certain point the person will have no effect on the solution and the solution for the other two persons will be determined by applying the hypothesis for the two person case to their situation.

The point where the person will not anymore be able to influence the solution will depend upon the possibility of the person to offer an alternative solution to the two person exchange situation between the other two persons, which is more favorable to one of these two persons and at the same time causes no loss to the person himself. In general this point will change if the utility functions of the persons change.

If neither a unique price is necessary nor agreement between all persons, the hypothesis is analog to the previous case. The solution will in general be different, because the relevant set S will be different. As the sets S of the previous case are included in the sets S of the present case, the sum of the utility gains of all persons in the previous case can be at most equal to the sum of the utility gains of the present case.

The hypothesis may again be tested by playing a three person game with similar characteristics as the two person game. Finally a graphic solution of the competitive three person two commodity isolated exchange situation is shown.

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